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Welcome to YES Family!!

To begin with, we endorse our heartfelt thank you for showing your trust and confidence in YES Academy. We take pride to welcome you to this prestigious Academy, foundations of which are based on commitment, quality education and integrity. It has been our constant endeavor to deliver better and better. In our attempt to achieve mark of excellence and beyond, we would be even more grateful to have received your continued faith and love. We assure you, your trust will not go in vain and as reflected by our Vision Statement, we would continue to produce top achievers and successful professionals in Chartered Accountancy, Company Secretary, Cost & Management Accountancy and Law Entrance courses as we have been doing for almost a decade now.

Combined experience of Team YES is 80+ years and adding value each day. We have delivered outstanding results in the past with a bouquet of All India Rankers across all the levels of CA, CS, CMA and Law entrance examinations, and with your efforts, we are confident we will grow together.

Student convenience has always occupied a centre place at YES Academy and we strive to improve ourselves each day as we sincerely believe that improvement always has its own space, no matter what. Any suggestions from you are always welcome. Though Team shares a very good rapport with all of its students and the students feel very comfortable talking to any of their Teachers, still, if you wish to send us a suggestion, please feel free to write to us yesacademypune@gmail.com or get in touch with us at 8888 235 235 /8888 545 545.

We assure you the best of success and pride. And yes, it's not just a bond of your course duration, but a relationship for life now. We welcome you in advance to this prestigious journey of professional excellence.

On behalf of TEAM YES

CS VIKAS VOHRA
(Founder)

CA CS HARISH A. MATHARIYA
(Founder)

Business Mathematics & Statistics

- CS Adv Vaibhav Chitlangia

YES ACADEMY

अविचार्य तु कर्माणि सिद्धिं यान्ति साधकाः।
धैर्येण सततं कर्म यथा सिद्धिं न गच्छति ॥

"Without doubt, those who persistently engage in their actions with determination to achieve success. It is through consistent effort & courage that success is attained."

I dedicate this book to -

My Wife - Paridhi

without whom this book couldn't have been possible!

Wishing you all the best of luck!

God Bless!

Vaibhav

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Ch 1.1 – Ratio, Proportion & Variations

A. Ratios

A ratio is a comparison of two or more quantities that are of the same kind and same unit.

Ratios can be shown in form of a fraction (p/q) or in the form of an expression using $(:)$ symbol.

Eg. – If $a = 5kg$ and $b = 10kg$, then the ratio of a and be can be shown as

$$a : b = \frac{a}{b} = 5 : 10 = \frac{5}{10}$$

Here, a and b are called the terms of the ratio. While a (the first term) is called the antecedent, b (the second term) is called the consequent.

Properties of Ratios

i) Both the terms of a ratio can be

divided and multiplied by the same (non zero) number.

ii) Order of the terms of a ratio is important.

iii) Ratios can exist only between quantities of the same kind.

iv) In order to compare different ratios, they have to be first converted into like fractions i.e; the consequent of both ratios are required to be made equal.

v) Ratios do not have any unit.

RATIOS IN THE SIMPLEST FORM

When both the terms of a ratio are reduced to such a form that they can not be further divided by any same number, then such a form is known as the simplest form of the ratio.

Eg. -

		Simplest form
$\frac{5}{10}$	$\rightarrow$	$\frac{1}{2}$
$\frac{8}{40}$	$\rightarrow$	$\frac{1}{5}$

INCREASE / DECREASE IN QUANTITY

If a quantity is either increased or decreased in the ratio a : b, then the new quantity can be found by -

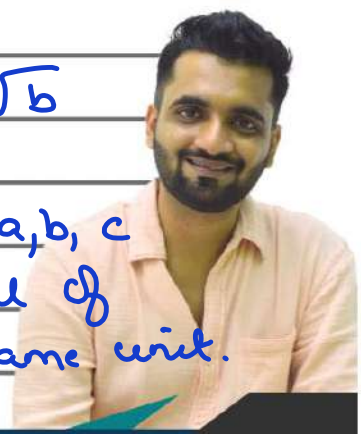
$$\text{New Quantity} = \text{Original Quantity} \times \frac{b}{a}$$

$$\text{Original Quantity} = \text{New Quantity} \times \text{Given Ratio}$$

In such cases, the ratio by which the original quantity is increased / decreased is called the factor multiplying ratio.

Types Of Ratios

Type	Ratio	Answer
Inverse Ratio	$a : b$	$b : a$
Ratio of Greater Inequality	$a : b$	If $a > b$
Ratio of Lesser Inequality	$a : b$	If $a < b$
Ratio of Equality	$a : b$	If $a = b$
Ratio Compounded	$a : b$ & $c : d$	$ac : bd$
Duplicate Ratio	$a : b$	$a^2 : b^2$
Triplicate Ratio	$a : b$	$a^3 : b^3$
Sub Duplicate Ratio	$a : b$	$\sqrt{a} : \sqrt{b}$
Sub Triplicate Ratio	$a : b$	$\sqrt[3]{a} : \sqrt[3]{b}$
Continued Ratio	$a : b : c$	When a, b, c are all of the same unit.



EXERCISE

B. PROPORTION

An equality of two ratios is called a proportion. It means that when two ratio follow a certain rule of equality, then they are said to be in proportion.

* Two ratios $a:b$ and $c:d$ are in proportion if

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b :: c:d$$

In the above proportion, a, b, c and d are called terms of the proportion.
The 1st and 4th terms of a proportion are called extreme terms.
The 2nd and 3rd terms of a proportion are called mean terms.

NOTE →
Since a proportion is made up of two ratios, it is NOT MANDATORY for all the four terms to be of the same unit.

Cross Product Rule

According to this rule, if a, b, c, d are the terms of a proportion such that

then

$$a : b :: c : d$$

$$a \times d = b \times c$$

ie product of extreme terms = product of mean terms.

Continued Proportion

Three terms a, b, and c are said to be in continued proportion if

$$a : b :: b : c$$

$$\frac{a}{b} = \frac{b}{c}$$

$$a \times c = b^2$$

$$b = \sqrt{a \times c}$$

Here, b is known as the mean proportion of a & b.

Properties Of Proportions

Invertendo

If $a:b = c:d$ then, $b:a = d:c$

Alternendo

If $a:b = c:d$ then, $a:c = b:d$

Componendo

If $a:b = c:d$ then, $a+b:b = c+d:d$

Dividendo

If $a:b = c:d$ then, $a-b:b = c-d:d$

Componendo & Dividendo

If $a:b = c:d$ then,

$a+b : a-b = c+d : c-d$

Addendo

If $a:b = c:d = e:f \ldots = k$, then
$(a+c+e \ldots) : (b+d+f+\ldots) = k$.

SUBTRAHENDO

If $a : b = c : d = e : f = k$

then

$(a - c - e \ldots) : (b - d - f \ldots) = k$

ADDENDO & SUBTRAHENDO

If $a : b = c : d$

then,

$$\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{a-c}{b-d}.$$

EXERCISE

C. VARIATIONS

Variations deals with cases where two variables x and y are so related that any change in one variable (x) results in some change in the other variable (y). Variations can be of two types -

i) DIRECT → When an increase / decrease in one variable results in a simultaneous increase/ decrease in the other variable.
It is shown using the sign $\propto$.
Eg. - Circumference $\propto$ radius.
$\Rightarrow C \propto r \qquad \Rightarrow C = kr$
$\Rightarrow k = \frac{C}{r}$

K is the constant of proportionality.

ii) INDIRECT → When an increase / decrease in one variable results in a simultaneous decrease/ increase in the other variable. Both variables are shown as reciprocals using the sign $\propto$.
Eg. - Speed $\propto \frac{1}{\text{time}}$

$\Rightarrow sp = k \times \frac{1}{t}$

$\Rightarrow k = s \times t$

JOINT VARIATION

If one variable varies directly with the product of other variables, then it is known as joint variation.

Eg. - Volume of cuboid $\propto$ length x breadth x height

$\Rightarrow V \propto l \times b \times h$

$\Rightarrow V = k \times lbh$

$\Rightarrow k = \frac{V}{lbh}$

ELEMENTARY RULES.

i) If $A \propto B$, then $B \propto A$

ii) If $A \propto B$ & $B \propto C$, then $A \propto B$

iii) If $A \propto B$ & $B \propto C$, then $A - B \propto C$

iv) If $A \propto C$ & $B \propto C$, then $A - B \propto C$

v) If $A \propto C$ & $B \propto C$, then $\sqrt{AB} \propto C$

vi) If $A \propto B$, then $A^n \propto B^n$

vii) If $A \propto B$ & $C \propto D$, then $AC \propto BD$ & $\frac{A}{C} \propto \frac{B}{D}$

viii) If $A \propto BC$, then $B \propto \frac{A}{C}$ & $C \propto \frac{A}{B}$.

EXERCISE

Ch 1.2 Time Value of Money

Mathematics of finance is a topic that encompasses concepts related to modes of funding for any entity like simple interest, compound interest, annuity and their application.

Important Definitions

- **Principal (P)** → The amount that is borrowed or invested. The initial amount of funding.
- **Interest (I)** → The extra amount that is paid over and above the principal which is the fees for use of principal.
- **Rate of Interest (R)** → The percent at which interest is charged on the principal.
- **Time Period (T)** → The time for which the principal amount is kept with the borrower / investee. Interest is charged for this time period.
- **Amount (A)** → The total amount repaid by the borrower / investee that includes principal and interest.

Why is Interest Charged?

- Time value of money
- Opportunity Cost
- Inflation
- Liquidity
- Risk Factors.

SIMPLE INTEREST

Interest that is computed on the outstanding Principal amount only and not on outstanding accumulated Interest. It is denoted by SI.

$$\boxed{SI = \frac{P \times R \times T}{100}}$$

where,

P = Principal
R = Rate of Interest
T = Time (in years)

Also,

$$\boxed{A = P + SI}$$

Calculator Trick for SI questions

To find SI → Press P

× i

× T (in years)

$i = \frac{R}{100}$.

To find P → Press SI

÷ T

÷ i

To find T → Press SI

÷ i

÷ P

To find R → Press SI

÷ P

÷ T

× 100

EXERCISE

COMPOUND INTEREST

The concept where interest is calculated on the principal as well as the accumulated interest of the past periods. It is denoted by CI.

$$A = P \times \left[1 + \frac{R}{100}\right]^T.$$

$$CI = A - P$$
$$= P \times \left[1 + \frac{R}{100}\right]^T - P$$

Generally, R is given on an annual basis and the CI is also calculated annually.
The collective period for which interest is calculated is called the Conversion Period.

For CI questions, Conversion Periods can be of different types -

- annually
- half yearly
- monthly
- quarterly.

For a given question where time is (t) and rate is (r), then the following operations have to be done-

When interest is compounded		
Annually	$R = r$	$T = t$
Half Yearly	$R = r/2$	$T = 2t$
Quarterly	$R = r/4$	$T = 4t$
Monthly	$R = r/12$	$T = 12t$

$$A = P \times \left[1 + \frac{R}{100}\right]^T.$$

Calculator Tricks for CI Questions

Basic Method to find A

→ Press P
→ + R
→ %

} Repeat upto n times

$n = T$.

To find CI, subtract P from above.

Short cut Method.

To find A (FV) when P is given –

→ Press $1+i$ $\quad i = \frac{R}{100}$.
→ ×
→ = = upto $\boxed{n+1}$
→ × P

To find P (PV) when A is given –

→ Press $1+i$ $\quad i = \frac{R}{100}$
→ ÷
→ = = upto $\boxed{n+2}$
→ × A

Present Value & Future Value

$$P = \frac{A}{(1+i)^n}$$

$$A = P(1+i)^n$$

where,

A = Amount = Future Value

P = Principal = Present Value

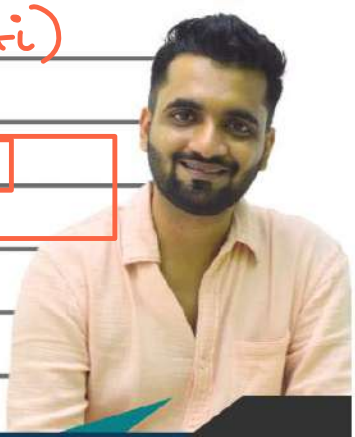
$i = \frac{R}{100}$

R = Rate of Interest

n = Time Period

Calculator Trick for PV & FV.

Present Value	Future Value
→ Press $(1+i)$	→ Press $(1+i)$
→ ÷	→ ×
→ = = $\boxed{n+2}$	→ = = $\boxed{n+1}$
→ × A.	→ × P



EXERCISE

Effective Rate of Interest

In cases where compound interest is charged in ways other than being compounded annually, effective rate of interest is calculated to find out the rate of interest effective on the transaction if interest was charged annually.

Trick to find Effective Rate of Interest

→ Calculate (R) and (T) by giving effect of conversion periods.

On the calculator, follow these steps –

→ Press $(1+i)$ $\qquad i = \frac{R}{100}$.

→ $\times$

→ $= \; = \; \boxed{n+2}$

→ $\times P$

→ $- 100$

The number of the screen is the effective rate of interest.

* For these questions, always take the Principal as 100 and time as 1 year. $(P=100, \; T=1)$.

EXERCISE

ANNUITY.

An annuity is a series of recurring cash payments that occur at regular intervals. Every annuity must have the following features -

a) payments made must be the same
b) time interval b/w two payments must be constant.

ANNUITY.

- Regular / Ordinary → Payments / Receipts happen at the end of each period.
- Due → Payments / Receipts happen at the beginning of every period.

ANNUITY

- PRESENT VALUE
  Eg. - Loans
  Borrowings
- FUTURE VALUE
  Eg. - Savings
  Sinking Fund
  Investment

ANNUITY REGULAR

PRESENT VALUE

$$A = P\left[\frac{(1+i)^{-n}-1}{i}\right]$$

FUTURE VALUE

$$A = P\left[\frac{(1+i)^{n}-1}{i}\right]$$

TRICK

→ Press $(1+i)$
→ ÷
→ = = $\boxed{n+2}$

→ GT
→ × P

→ Press $(1+i)$
→ ×
→ = = $\boxed{n+1}$

→ − 1
→ ÷ i
→ × P

If your calculator does not have 'GT',

→ Press $(1+i)$
→ ÷
→ = = $\boxed{n+2}$

→ − 1
→ ÷ i
→ × P

(ignore the −ve sign)

ANNUITY DUE

PRESENT VALUE

$$A = P\left[\frac{(1+i)^{-n}-1}{i}\right]\times(1+i)$$

FUTURE VALUE

$$A = P\left[\frac{(1+i)^{n}-1}{i}\right]\times(1+i)$$

TRICK

→ Press $(1+i)$
→ $\div$
→ $==$ $\boxed{n+2}$
→ GT
→ $\times P$
→ $\times(1+i)$

→ Press $(1+i)$
→ $\times$
→ $==$ $\boxed{n+1}$
→ -1
→ $\div i$
→ $\times P$
→ $\times(1+i)$

where,

A = Annuity Amount
P = Periodic Payment
$i = \frac{R}{100}$
R = Rate of Interest
n = no. of conversion periods.

AMORTIZATION

The process of gradually paying off a debt over a specific time through payment regular instalments.

* This method is used to find out the amount of periodic payments (EMIs) to be made to pay off a loan where the loan amount & interest rate is given.

→ Press $(1+i)$
→ ÷
→ = = $\boxed{n+2}$

→ −1
→ ÷ i } OR GT
→ ÷
→ =
→ Amount
→ =

* If the EMIs are given and the loan amount is to be found, use the trick for PV of Annuity.

SINKING FUND

Setting aside of money over a period of time to accumulate a specified amount for a future obligation.

* To find the amount that has to be kept aside periodically to accumulate a specified amount over a specific time at a specific rate of interest, use this method.

→ Press $(1+i)$
→ $\times$
→ $= =$ $\boxed{n+1}$

→ -1
→ $\div i$
→ $\div$
→ $=$
→ Amount
→ $=$

Here 'Amount' is the money that is required to be accumulated in future.

Application of Annuity.

The principles of annuity are applied in solving questions related to

- → Leasing
- → Capital Expenditure
- → Valuation of Bond.

EXERCISE

Ch 1.3 – Arithmetic Progression & Geometric Progression

A collection of numbers that follow the same pattern as per some definite rule is called a progression

Eg – 2, 4, 6 8 .....

Arithmetic Progression

A sequence is said to be an Arithmetic Progression (A.P.) if each term is obtained by adding a constant value (d) to the previous term.

This constant value (d) is known as the Common Difference.

Therefore, in an AP with the terms $a_1, a_2, a_3, a_4 \ldots$

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = d.$$

To find the nth term of an AP -

$$\boxed{a_n = a + (n-1)d.}$$

where,

a_n = n^{th} term
a = 1st term
n = no. of terms
d = common difference

Sum Of Terms Of AP.

- Sum of 1st n terms

$$\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{OR} \quad S_n = \frac{n}{2}(a + l)}$$

where,

a = 1st term
l = Last term
n = no. of terms
d = Common difference

SPECIAL CASES.

- Sum of 1st n natural / counting numbers

$$\boxed{S_n = \frac{n(n+1)}{2}}$$

- Sum of 1st n odd numbers

$$\boxed{S_n = n^2}$$

- Sum of squares of 1st n natural numbers

$$\boxed{S_n = \frac{n(n+1)(2n+1)}{6}}$$

- Sum of cubes of 1st n natural numbers

$$\boxed{S_n = \left\{\frac{n(n+1)}{2}\right\}^2}$$

EXERCISE

GEOMETRIC PROGRESSION

A sequence of terms where each term is a constant multiple of the previous term is known as a Geometric Progression (G.P.)

The multiplier is known as Common Ratio (r).

Eg. = $\underset{a}{2}$, $\underset{a \times r}{4}$, $\underset{a \times r^2}{8}$, $\underset{a \times r^3}{16}$, $\underset{a \times r^4}{32}$ ....

$$\boxed{\text{Common Ratio } (r) = \frac{\text{Any term}}{\text{Preceeding Term}}}$$

n^{th} term of a GP.

To find the nth term of a GP,

$$\boxed{t_n = a \times r^{n-1}}$$

where,

a = 1st term

r = common ratio

n = no. of terms.

Geometric Mean

When the terms a, b and c are in GP, then

$$\frac{b}{a} = \frac{c}{b}$$

$$\therefore \boxed{b^2 = ac}$$

Here b is called the geometric mean of a and c.

Sum Of The Terms Of A G.P.

- If common ratio, $r > 1$ then

$$\boxed{S_n = \frac{a(r^n - 1)}{r - 1}}$$

- If common ratio, $r < 1$ then

$$\boxed{S_n = \frac{a(1 - r^n)}{1 - r}}$$

- If common ratio, $r = 1$

$$\boxed{\begin{aligned} S_n &= a + a + a + \dots a \text{ (n terms)} \\ S_n &= n \times a. \end{aligned}}$$

- If the GP is an infinite series

$$\boxed{S_n = \frac{a}{1 - r}}$$

where $r < 1$

EXERCISE

Ch 1.4 - TIME & DISTANCE

Time and Distance have a mathematical relationship with speed where speed is said to be the ratio of distance and time.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Speed}}{\text{Distance}}$$

$$\text{Distance} = \text{Speed} \times \text{time}.$$

Generally, speed is denoted in either km/hour or m/sec.

To convert –

$$\text{Km/hr} \times \frac{5}{18} = \text{m/sec}$$

$$\text{M/sec} \times \frac{18}{5} = \text{km/hour}.$$

Exercise

Ch 2.1 – Set Theory & Venn Diagram

A Set is a collection of well defined distinct objects. It is generally depicted in the following manner –

$$A = \{a, e, i, o, u\}$$

where,

A is the Set

a, e, i, o, u are the elements.

NOTE →

i) Order of elements in a Set is of NO IMPORTANCE

ii) Repetition of elements is also of NO IMPORTANCE

Eg. → $A = \{1, 1, 2, 5, 6, 6, 8, 9\}$

$B = \{1, 5, 8, 9, 6, 2\}$

Set A = Set B because order / elements will have no effect !!

Forms Of Sets

Sets can be presented in 2 forms –

Roster / Braces

$A = \{a, e, i, o, u\}$

Rule / Algebraic

$A = \{x : x \text{ is a vowel}\}$

Relevant Symbols

- $\in$ = belongs to
- $\notin$ = does not belong to

$a \in A$ means a is an element of set A.
$b \notin A$ means b is NOT an element of set A.

Cardinal Number

The number of elements in a set is known as the Cardinal Number of the set. It is denoted by 'n'.

$A = \{a, e, i, o, u\}$ then, $nA = 5$.

Types of Sets

- **Finite Set** → The type of set that contains a finite number of elements.
  $A = \{a, e, i, o, u\}$.

- **Infinite Set** → The type of set that contains an infinite number of elements.
  $A = \{x : x \text{ is a natural number}\}$

- **Null / Empty Set** → The type of set that contains NO elements. It is denoted by ϕ or $\{\}$.
  $A = \phi$

- **SINGLETON SET** → A set that contains ONLY 1 element.
  $A = \{a\}$

- **EQUAL SETS** → Two sets are called equal if they have the exact same elements.
  $A = \{a, e, i\}$ $B = \{e, i, a\}$

- **EQUIVALENT SETS** → Two sets that have the same number of elements i.e; the same cardinal number.
  $A = \{a, e, i\}$ $B = \{i, o, u\}$

* Every equal set is equivalent but not every equivalent set is equal.

- **DISJOINT SET** → Two sets that do not have any common element.
  $A = \{a, e\}$ $B = \{i, o\}$

- **UNIVERSAL SET** → A set that contains all the elements under study.
  It is denoted by 'U'
  $U = \{x : x \text{ is an alphabet}\}$

- **COMPLEMENTARY SET** → A set is called complementary when it contains all the elements of the Universal set

that are not present in the original set.
It is denoted by Set' or Set^c.

$U = \{a, e, i, o, u\}$

$A = \{a, e, i\}$ $\qquad$ $A' = \{o, u\}$

SUBSET & SUPERSET

A set A is called the subset of another set B if all the elements of set A are present in set B.

In such cases, A is called the subset and B is called the Superset.

$B = \{a, e, i, o, u\}$ – Superset

$A = \{a, e, i\}$ – Subset

Subsets are of two types

PROPER

A subset that does not contain all the elements of the super set.

IMPROPER

A subset that contains all the elements of the superset.

$A = \{a, e, i, o, u\}$

$C = \{a, e, i, o, u\}$ $\qquad$ $B = \{a, e, i\}$

$C \subseteq A$ $\qquad$ $B \subset A$

Total no. of Subsets

For a superset that contains 'n' no. of elements, the total no. of subsets can be found by

$$\boxed{\text{No. of Subsets} = 2^n}$$

$$\boxed{\text{No. of Proper subsets} = 2^n - 1}$$

$$\boxed{\text{No. of Improper subsets} = 1.}$$

POWER SET.

A set that contains all possible subsets of a given set is called the power set. For a given set A, the power set will be denoted by P(A).

PROPERTIES OF SUBSETS

- If $A \subseteq B$ and $B \subseteq A$, then $A = B$
- Every set is a subset of itself. $A \subseteq A$
- The empty set (ϕ) is always a subset of any set.

LAWS OF ALGEBRA OF SETS

1. Idempotent

$$A \cap A = A$$
$$A \cup A = A$$

2. Commutative

$$A \cap B = B \cap A$$
$$A \cup B = B \cup A$$

3. Associative

$$A \cup (B \cup C) = (A \cup B) \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C$$

4. Distributive

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

5. De Morgan's Law

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$

6. Identity Law

$A \cup U = U$

$A \cup \phi = A$

$A \cap \phi = \phi$

$A \cap U = A$

7. Complement Law

$A \cup A' = U$

$\phi' = U$

$U' = \phi$

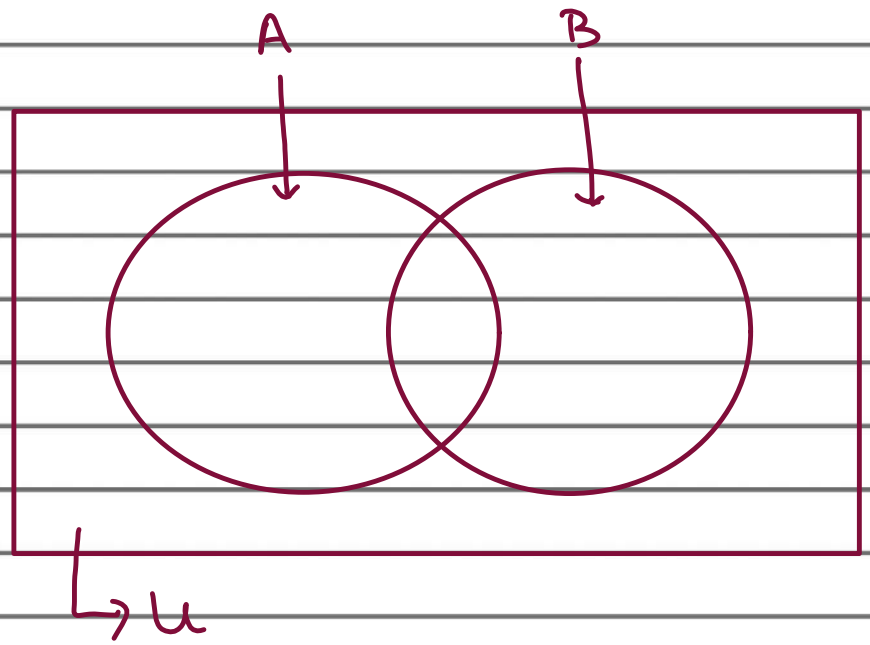
$A \cap A' = \phi$

$(A')' = A$

VENN DIAGRAMS

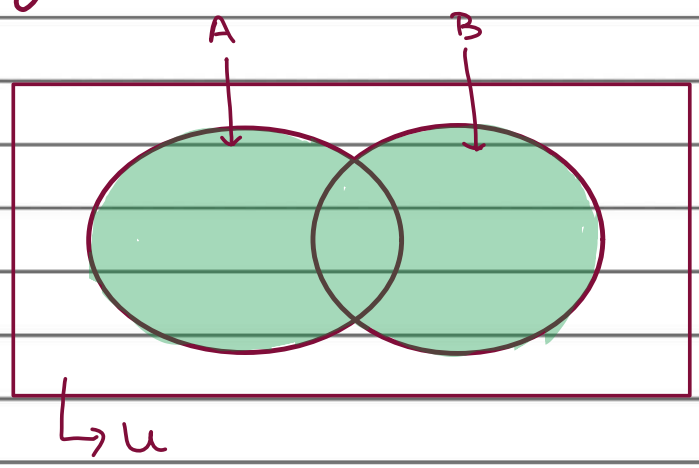
A Venn Diagram is a graphical representation of relationships between two sets. Following are some important principles -

- The Universal Set is represented by a rectangle.
- The subsets of the Universal Set are represented by using circles inside the rectangle.
- The points where the circles overlap indicate the elements that are common to both subsets.
- The area where the circles DO NOT overlap represent elements that are NOT COMMON to the given sets



OPERATIONS OF VENN DIAGRAMS

- **Union of Sets**



$A \cup B$

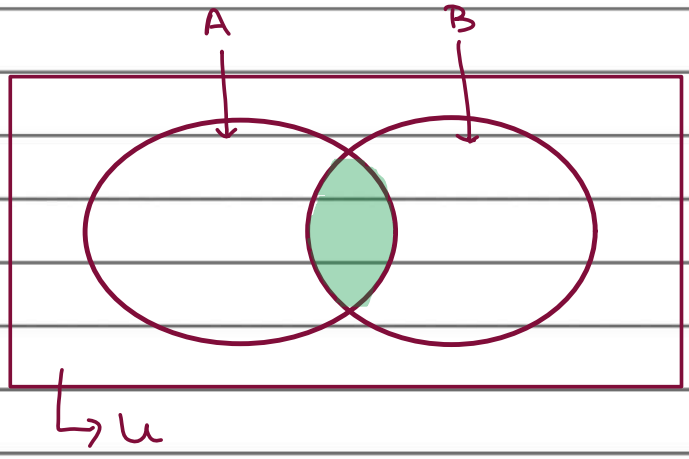
$A \cup B$ represents the set containing all those elements that are present in Set A and Set B, common elements being taken only one.

$A = \{a, e, i, o, u\}$

$B = \{a, b, c, d, e\}$

$A \cup B = \{a, b, c, d, e, i, o, u\}$

- Intersection of Sets



$A \cap B$

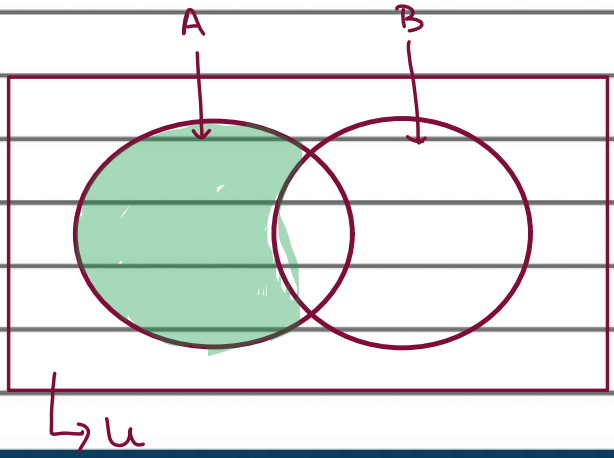
$A \cap B$ represents those elements that are common to both sets A & B.

$A = \{a, e, i, o, u\}$
$B = \{a, b, c, d, e\}$

$A \cap B = \{a, e\}$.

- Difference of two Sets.

$A - B$

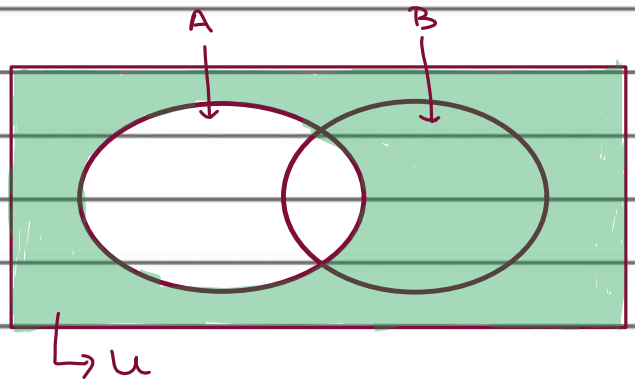


A-B represents those elements that are present in Set A but not in Set B.

$$A = \{a, e, i, o, u\}$$
$$B = \{a, b, c, d, e\}$$

$$A - B = \{i, o, u\}.$$

- Complement of a Set

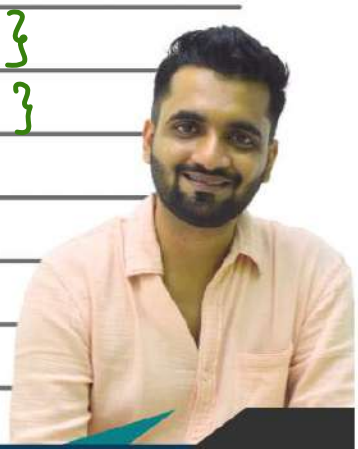


A' or A^c

A' or A^c is the set that contains all the elements that are there in the Universal Set but not in set A.

$$U = \{x : x \text{ is an alphabet}\}$$
$$A = \{y : y \text{ is a consonant}\}$$

$$A' = \{a, e, i, o, u\}.$$



Cardinal Number In Different Sets.

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

Products of Sets

If A and B are two sets such that $a \in A$ and $b \in B$, then the product of $A \times B$ that is in the form (a, b) are called Cartesian Product of sets.

$A = \{1, 2\}$
$B = \{3, 4\}$

$A \times B = \{1,3\}\ \{1,4\}\ \{2,3\}\ \{2,4\}$

Note = $A \times B \neq B \times A$

EXERCISE

Ch 2.2 - Indices & Logarithms

Calculator Tricks

To find the nth power of x (x^n)

→ Type x
→ Press $\times$
→ Press $= =$ upto $\boxed{n+1}$

To find the nth root of x $(\sqrt[n]{x})$

→ Type x
→ Press $(\sqrt{\ })$ 12 times
→ -1
→ $\div n$
→ $+1$
→ Press $(\times)$ & $(=)$ → 12 times

A. INDICES

Index is the number that tells us how many times a number has been multiplied by itself. It is also often referred to as 'power'. The plural of this term is the word 'Indices'.

Eg. - $7^4 = 7 \times 7 \times 7 \times 7$.

In this example,

7 is the Base
4 is the Index.

Basic Rules of Indices

- **Rule 1**

$$a^m \times a^n = a^{(m+n)}$$

- **Rule 2**

$$(a^m)^n = a^{mn}$$

- **Rule 3**

$$\frac{a^m}{a^n} = a^{m-n}$$

- **Rule 4**

$$a^{-m} = \frac{1}{a^m}$$

- **Rule 5**

$$a^0 = 1 \quad (\text{where } a \neq 0)$$

- **Rule 6**

$$a^{1/m} = \sqrt[m]{a}$$

- **Other Rules**

(i) $(ab)^m = a^m \times b^m$

(ii) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(iii) $a^{m/n} = \sqrt[n]{a^m}$

EXERCISE

CALCULATOR TRICKS

To find the nth power of x (x^n) when n is in decimals

→ Type 'x'
→ Press '$\sqrt{\ }$' 12 times
→ -1
→ $\times (n)$
→ $+1$
→ Press $(\times)$ & $(=)$ 12 times

To find Log x

→ Type x
→ Press '$\sqrt{\ }$' 12 times
→ -1
→ Multiply by 14230.9635

B LOGARITHMS

In the expression

$$\log_a x = b$$

'b' is the power to which 'a' must be raised to get the number 'x'.

$$[a^b = x]$$

Thus, logs are used to find the index for a given base and solution.

Eg.- $\log_3 9 = 2$ $\quad 3^2 = 9$

$\log_{10} 100 = 2$ $\quad 10^2 = 100$

$\log_3 27 = 3$ $\quad 3^3 = 27$

$\log_3 1 = 0$ $\quad 3^0 = 1$

* In case the expression is in the form of $\log x = b$, then the base is always taken as 10

[10 → Common base of logs].

ANTILOGARITHMS

If $\log x = n$

Then $x = \text{Antilog } n$.

FUNDAMENTAL LAWS OF LOGARITHMS

- RULE 1

$$\log_b a = \frac{\log a}{\log b}$$

$$\log_3 2 = \frac{\log 2}{\log 3}$$

- RULE 2

$$\log_a mn = \log_a m + \log_a n$$

$$\log_a(10) = \log_a(5\times 2) = \log_a 5 + \log_a 2$$

- **Rule 3**

$$\log_a\left[\frac{m}{n}\right] = \log_a m - \log_a n$$

$$\log_a\left[\frac{27}{5}\right] = \log_a 27 - \log_a 5$$

- **Rule 4**

$$\log_a m^n = n\log_a m$$

$$\log_2 2^4 = 4\log_2 2$$

- **Rule 5**

$$\log_a a = 1$$

$$\log_{10} 10 = 1 \quad , \quad \log_2 2 = 1.$$

- **Rule 6**

$$\log_a 1 = 0$$

$$\log_6 1 = 0 \qquad \log_{100} 1 = 0$$

- **Rule 7**

$$\log_b a \times \log_a b = 1$$

$$\log_3 2 \times \log_2 3 = 1$$

- **Rule 8**

$$\log_b a \times \log_c b = \log_c a$$

$$\log_3 2 \times \log_4 3 = \log_5 2$$

- **Rule 9**

$$\log_b a = \frac{1}{\log_a b}$$

$$\log_2 3 = \frac{1}{\log_3 2}$$

- **Rule 10**

$$\log_b m \times \log_a b = \log_a m$$

$$\log_b m = \frac{\log_a m}{\log_a b}$$

EXERCISE

Ch 2.3 - Permutations & Combinations

Fundamental Principles of Counting

- **Addition Rule** → If there are two alternative jobs which can be done in 'm' ways & in 'n' ways respectively, then either of two jobs can be done in (m+n) ways.

- **Multiplication Rule** → If a certain thing may be done in 'm' different ways and once it is done, a second thing can be done in 'n' ways, then the total number of ways of doing both things is (m×n).

Eg. -

i) A wants to go to school. There are 5 buses & 4 autos that can take him to school.

∴ Total ways in which A can reach school is 4+5 = 9.

ii) A can go to school by 5 different buses and come back by 4 different autos.

∴ Total ways = 4×5 = 20.

THE FACTORIAL

The factorial n, which can be written as $n!$ or $\lfloor n$ means the product of all integers from 1 to n, both inclusive.

$$\boxed{n! = n \times (n-1) \times (n-2) \cdots \times 3 \times 2 \times 1}$$

$$\boxed{\text{NOTE} \rightarrow 0! = 1.}$$

5!

PERMUTATION

The ways of arranging or selecting smaller or equal number of persons or objects from a group of persons with due regard being paid to the order of selection are called permutations.

Permutations are denoted by

^nP_r OR nP_r OR $P_{n,r}$.

where

n = Total no. of objects
r = No. of objects to be arranged.

Permutations are found using the following formula

$$\boxed{^nP_r = \frac{n!}{(n-r)!}}$$

CIRCULAR PERMUTATIONS

- If 'n' different things are to be arranged in a circle, then the number of ways in which such an arrangement can be done is

$$\boxed{{}^nP_r = (n-1)!}$$

- No. of ways of arranging 'n' people around a table so that no two person has the same two neighbours is

$$\boxed{{}^nP_r = \frac{1}{2}(n-1)!}$$

- No. of necklaces that can be formed with 'n' type of beads

$$\boxed{{}^nP_r = \frac{1}{2}(n-1)!}$$

Permutation With Restrictions

- No. of permutations of 'n' objects when a particular object is not taken in

$$\boxed{{}^{n-1}P_r}$$

- No. of permutations of 'n' objects when a particular object is always taken -

$$\boxed{{}^{n-1}P_{r-1}}$$

- Permutation when each 'n' thing is repeated 'r' times -

$$\boxed{P = n^r}$$

- Permutation when out of 'n' things, n_1 are alike, n_2 are alike and so on, when all 'n' things taken at a time

$$\boxed{P = \frac{n!}{n_1!\, n_2!\, n_3!}}$$

EXERCISE

COMBINATIONS.

The number of ways in which smaller number of things are arranged out of a collection of 'n' things such that the order of selection or arrangement is NOT important, are called Combinations.

Combinations are denoted by ${}^nC_r$.

$$\boxed{{}^nC_r = \frac{n!}{r! \times (n-r)!}}$$

PROPERTIES OF COMBINATIONS

i) If $r = n$, then $\boxed{{}^nC_r = {}^nC_n = 1.}$

ii) If $r = 0$, then $\boxed{{}^nC_r = {}^nC_0 = 1}$

iii) $\boxed{{}^nC_r = {}^nC_{n-r} \quad (\text{Eg} \Rightarrow {}^5C_4 = {}^5C_1)}$

iv) $\boxed{{}^{n+1}C_r = {}^nC_r + {}^nC_{r-1}}$

OTHER RULES.

- Combination of 'n' things taking some or all of 'n' things at a time

$$\boxed{C = 2^n - 1}$$

- No of handshakes in a given group of 'n' people

$$\boxed{\frac{n(n-1)}{2}}$$

- No. of diagonals in a Polygon with 'n' sides -

$$\boxed{{}^nC_2 - n}$$

- If there are 'm' points in a plane out of which 'a' are collinear, then

$$\boxed{\begin{array}{l} \text{No. of straigh lines} = {}^mC_2 - {}^aC_2 + 1 \\ \text{No. of Triangles} = {}^mC_3 - {}^aC_3 \end{array}}$$

EXERCISE

CH 2.4 - QUADRATIC EQUATIONS

An equation is a statement that supports the equality of two expressions. These expressions are connected by the '=' sign.

Eg - $2x + 3 = 15$.

CLASSIFICATION OF EQUATIONS

i) LINEAR → Highest power of variable is 01.
Eg - $x + 2 = 5$

ii) QUADRATIC - Highest power of variable is 02.
Eg - $x^2 + 3 = 7$

iii) CUBIC - Highest power of variable is 03.
Eg - $x^3 + 3 = 11$

Quadratic Equation

An equation where the highest power of the variable is 02 is known as a quadratic equation. It is in the form

$$ax^2 + bx + c = 0$$

where $a \neq 0$

In a quadratic equation,

if $b = 0$

Equation $= ax^2 + c = 0$

Pure Quadratic Eqn.

If $b \neq 0$

Eqn. $= ax^2 + bx + c = 0$

Affected Quadratic Eqn.

Every Quadratic Equation has two roots. ie two values of the variable that satisfy the equation.

Methods of Solving Quadratic Equations

i) Prime Factorization Method
ii) Quadratic Method

(i) PRIME FACTORIZATION METHOD

In this method, for an equation

$$ax^2 + bx + c = 0,$$

'bx' is broken into two components whose sum equals 'bx' and product equals '$ax^2 \times c$'.

Eg – $x^2 + 5x + 6 = 0$

$\Rightarrow x^2 + 3x + 2x + 6 = 0$

$x(x+3) + 2(x+3) = 0$

$(x+2)(x+3)$

$x = -3, -2.$

(ii) By Quadratic Formula

For a given equation

$$ax^2 + bx + c = 0,$$

root 'x' can be found by -

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Shridharacharya Formula.

Important Properties

i) If $b^2 - 4ac = 0$, roots are real & equal.

ii) If $b^2 - 4ac > 0$, roots are real & distinct.

iii) If $b^2 - 4ac < 0$, roots are imaginary.

iv) If $b^2 - 4ac$ is a perfect square and $\neq 0$, roots are rational & distinct.

v) If $b^2 - 4ac > 0$ but not a perfect square, roots are real, irrational and unequal.

vi) Irrational roots occur in conjugate pairs i.e. if one root is $(m + \sqrt{n})$, the other root is $(m - \sqrt{n})$.

Properties Of Roots

If α & β are roots of a quadratic equation, then

Sum of the Roots -

$$\alpha + \beta = \frac{-b}{a}$$

Product of the Roots

$$\alpha \times \beta = \frac{c}{a}$$

Equation →

$$x^2 - (\text{Sum of Roots})x + \text{Product of Roots} = 0.$$

- If $\alpha = \frac{1}{\beta}$, then their product $= 1$

  $\therefore \frac{c}{a} = 1 \Rightarrow c = a.$

- If $\alpha = -\beta$, then their sum $= 0$.

  $\therefore -\frac{b}{a} = 0 \Rightarrow b = 0.$

Important Identities

i) $(a+b)^2 = a^2+b^2+2ab$

ii) $(a-b)^2 = a^2+b^2-2ab$

$= (a+b)^2 - 4ab$

iii) $(a-b) = \sqrt{(a+b)^2 - 4ab}$

iv) $a^2-b^2 = (a+b)(a-b)$

v) $a^2+b^2 = (a+b)^2 - 2ab$

vi) $(a+b)^3 = a^3+b^3+3ab(a+b)$

vii) $(a-b)^3 = a^3-b^3+3ab(a-b)$

viii) $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

ix) $a^3-b^3 = (a-b)(a^2+ab+b^2)$

EXERCISE

CH 3.1 - CALCULUS & ITS APPLICATION

FUNCTION

If x and y are two variables such that for every value of x in a defined domain, we get a corresponding value of y, then y is said to be a function of x.

$$f(x) = F(x) = y.$$

TYPES OF FUNCTIONS

- **CONSTANT FUNCTION** - When y is the same for all values of x. Eg. - $y = f(x) = 7$
- **SINGLE VALUED FUNCTION** - When there exists a single value of y for every value of x. Eg - $y = f(x) = 2x+3$.
- **MULTI VALUED FUNCTION** - When there are more than one value of y for a value of x. Eg. - $y^2 = x$.
- **EXPLICIT FUNCTION** - When a function is expressed directly in terms of the independent variable. Eg. - $y = x^2 - x + 1$

- **IMPLICIT FUNCTION** – A function that is not explicit in nature.

  Eg. – $2x^2 + 3xy + y^2 = 0$

- **PARAMETRIC REPRESENTATION** – When both x & y are represented in terms of a third variable.

  Eg. – $y = t^2 + 1$, $x = 2t$.

- **ODD FUNCTION** – When $f(-x) = -f(x)$

- **EVEN FUNCTION** – When $f(-x) = f(x)$

- **INVERSE FUNCTION** – If from $y = f(x)$, $x = f(y)$ can be obtained, then both are inverse functions of one another.

  Eg. – $y = 4x + 3$ & $x = \frac{y-3}{4}$

- **POLYNOMIAL FUNCTION** – When a function is in the form $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots a_nx^n$.

- **RATIONAL FUNCTION** – A function expressed as a ratio of two polynomials ie;

$$f(x) = \frac{a_0 + a_1x + a_2x^2 + a_3x^3 \ldots\ldots + a_nx^n}{b_0 + b_1x + b_2x^2 + b_3x^3 \ldots\ldots + b_nx^n}$$

- **IRRATIONAL FUNCTION** – A function that can not be expressed as a ratio of two polynomials.

- **ALGEBRAIC FUNCTION** – A polynomial function that has a finite number of terms.

  Eg – $x^2 + 2x - 3$, $\sqrt{x^2+1}$

DOMAIN & RANGE

The set of value of 'x' (input) is called **DOMAIN**.

The set of corresponding values of 'y' (output) is called **RANGE**.

COMPLEX NUMBER

A number in the form $(a+ib)$ or $(a-ib)$ where a & b are real numbers and $i = \sqrt{-1}$ is called a complex number.

Eg. – $x^2 = -4$

$x = \sqrt{-4} = \pm 2\sqrt{-1} = \pm 2i$

Exercise

LIMITS

IMPORTANT DEFINITIONS

- x tends to a – when the difference $|x-a|$ can be made less than any positive quantity, however small. $[x \to a]$.

- x tends to zero – When the value of x goes on decreasing numerically and can be made less than any positive quantity, however small. $x \to 0$

- x tends to infinity – When the value of x goes on increasing numerically and can be made more than any positive quantity, however large. $x \to \infty$

METHODS OF FINDING LIMITS where $x \to a$

i) Method of factors
ii) Method of Substitution
iii) Method of Rationalization

EXERCISE

FUNDAMENTAL THEOREM OF LIMITS

If $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} \phi(x) = m$ where l and m are finite quantities, then –

i) $\lim_{x \to a} [f(x) \pm \phi(x)] = l + m.$

ii) $\lim_{x \to a} [f(x) \cdot \phi(x)] = l \times m$

iii) $\lim_{x \to a} \frac{f(x)}{\phi(x)} = \frac{l}{m}$ $\quad m \neq 0$

iv) If $\lim_{x \to a} \phi(x) = b$ and $\lim_{x \to b} f(u) = f(b)$ then

$$\lim_{x \to b} f\{\phi(x)\} = f\left\{\lim_{x \to b} \phi(x)\right\} = f(b).$$

v) $\lim_{x \to a} k = k$

vi) $\lim_{x \to a} k \cdot f(x) = k \lim_{x \to a} f(x)$

where k = constant.

EXERCISE

Some Useful Limits

i) $\lim_{x \to 1} (1+x)^{1/x} = e$

ii) $\lim_{x \to 0} \frac{1}{x} \log_e (1+x) = 1$

iii) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

iv) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

v) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$

vi) $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$

EXERCISE

CONTINUITY

A function is said to be continuous if its graph is a continuous curve without any breaks.
However, if there is any break in the graph, then the function is not continuous at that point.

A graph is said to be discontinuous when for a value of k, the limit of f(x) does not exist.

A function is continuous at $x = a$ when $\lim_{x \to a} f(x)$ exists, is finite, and is equal to $f(a)$.

To check whether a function is continuous, the following values must be equal -

i) $\lim_{x \to a+} f(x)$ ii) $\lim_{x \to a-} f(x)$ iii) $f(a)$

PROPERTIES OF CONTINUITY

i) Sum or difference of two continuous functions is a continuous function

ii) Product of two continuous functions is a continuous function.

iii) Ratio of two continuous function is a continuous function, where the denominator $\neq 0$.

CONTINUITY IN AN INTERVAL, AT END POINTS

A function is said to be continuous over the interval including the end points if it is continuous at every point of the interval.

Let c be any point in the interval (a,b) -

If $\lim_{x \to c+} f(x) = \lim_{x \to c-} f(x) = f(c)$,

then f(x) is continuous in the interval (a,b).

EXERCISE

DIFFERENTIAL CALCULUS

Differential calculus or differentiation helps in measuring change in one variable with respect to a small change in another variable.

With a small change in the value of 'x', the impact on the value of 'y' where

$$y = f(x)$$

is called the differential function of x on y.

It is shown by

$$\frac{dy}{dx} = \frac{\Delta x}{\Delta y} = y_1 = y' = f(x)'$$

↳ Slope of Tangent
↳ Gradient.

Basic Rule of differentiation -

$$\boxed{\frac{d(x^n)}{dx} = n(x)^{n-1} \quad \text{where } y = x^n}$$

Other Important Rules

i) $\frac{d(x^n)}{dx} = n(x)^{n-1}$

ii) $\frac{d(e^x)}{dx} = e^x \times \frac{dx}{dx} = e^x$

iii) $\frac{d(a)^x}{dx} = a^x \cdot \log a$

iv) $\frac{d(\log x)}{dx} = \frac{1}{x}$

v) $\frac{d(k)}{dx}$ (where k = constant) $= 0$

vi) $\frac{d(x)}{dx} = 1$

vii) $\frac{d(1/x)}{dx} = -\frac{1}{x^2}$

viii) $\frac{d(\log_a x)}{dx} = \frac{1}{x}\log_a e$

ix) $\frac{dy}{dx}$ where $y = y^2$

$\Rightarrow \frac{d(y^2)}{dx} = 2y \times \frac{dy}{dx}$

TRICK REGARDING CONSTANT

If the Constant is in

Addition / Subtraction → Write 0

Multiplication / Division → Write the number

Eg. = $x + 3$ → becomes 0

Eg. - $x^2 + 3x$ → becomes 3.

Basic Laws Of Differentiation

i) $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ (Sum Rule)

ii) $\frac{d(u-v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ (Difference Rule)

iii) $\frac{d(u \times v)}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$ (Product Rule)

iv) $\frac{d[u/v]}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$ (Quotient Rule)

v) $\frac{d(k \times f(x))}{dx} = k \times \frac{d f(x)}{dx}$

EXERCISE

DERIVATIVE OF FUNCTION OF A FUNCTION

This is used when a variable y is a function of a second variable z, which is a function of a third variable x.

If $y = f(z)$, $z = \phi(x)$ then $y = f\{\phi(x)\}$.

Also, $\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

Example → Find $\frac{dy}{dx}$ for $y = 2z^2+1$ & $z = 4x-2$.

$\frac{dy}{dz} = 4z$ $\qquad$ $\frac{dz}{dx} = 4.$

$$\therefore \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$
$$= 4z \times 4$$
$$= 16z$$
$$= 16(4x-2)$$
$$= 64x - 32$$

IMPLICIT FUNCTION

A function that is in the form of $(x, y) = 0$ is known as an implicit function.

Example.

Find $\frac{dy}{dx}$ when

$$x^2y^2 + 3xy + y = 0.$$

$$\Rightarrow x^2 \frac{dy^2}{dx} + y^2 \frac{dx^2}{dx} + 3x \frac{dy}{dx} + 3y \frac{dx}{dx} + \frac{dy}{dx}$$

$$\Rightarrow 2yx^2 \frac{dy}{dx} + 2xy^2 + 3x \frac{dy}{dx} + 3y + \frac{dy}{dx}$$

$$\Rightarrow (2yx^2 + 3x + 1) \frac{dy}{dx} + 2xy^2 + 3y.$$

$$\Rightarrow (2yx^2 + 3x + 1) \frac{dy}{dx} = -2xy^2 - 3y$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(2xy^2 + 3y)}{2yx^2 + 3x + 1}.$$

EXERCISE

PARAMETRIC EQUATION

When two variable x & y are expressed in terms of a third variable t, then the equation is called parametric equation.

EXAMPLE - Find $\frac{dy}{dx}$ when -

$x = at^3$ $\qquad$ $y = \frac{a}{t^3} = at^{-3}$

$$\frac{dx}{dt} = \frac{d(at^3)}{dt} = 3at^2$$

$$\frac{dy}{dt} = \frac{d(at^{-3})}{dt} = -3at^{-4} = \frac{-3a}{t^4}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{-\cancel{3}\cancel{a}/t^4}{\cancel{3}\cancel{a}t^2} = \frac{-1}{t^6}$$

$$= \underline{-t^{-6}}$$

EXERCISE

HIGH ORDER DIFFERENTIATION

This method is used in cases where a given variable can be differentiated multiple times.

Example -

$$y = f(x) = x^4 + 5x^3 + 2x^2 + 9$$

$$\frac{dy}{dx} = f'(x) = 4x^3 + 15x^2 + 4x + 0.$$

$$\frac{d^2y}{dx^2} = f''(x) = 12x^2 + 30x + 4$$

$$\frac{d^3y}{dx^3} = f'''(x) = 24x + 30$$

EXERCISE

Ch 3.2 Revenue & Cost Function
3.3 Optimization Techniques

Revenue means the total receipts obtained by a company through sale of commodities.

Total Revenue
Total amount of money that a company receives through sale of products

$TR = P \times Q$

P = Price
Q = Quantity

Marginal Revenue
Net Revenue earned by selling one additional unit of product produced.

$MR = TR_{n+1} - TR_n$

Average Revenue
Ratio of total Revenue to total number of units sold.

$$AR = \frac{TR}{Q} = \frac{P \times Q}{Q}$$

$AR = P.$

Marginal Revenue can also be expressed as

$$MR = \frac{d(TR)}{dQ}$$

EXERCISE

Cost

Total Cost is the sum of all costs incurred by a firm in producing a certain level of output.

Total Fixed Cost
Costs that do not vary with output. (TFC)

Total Variable Cost
Costs that change with the level of output (TVC)

$$TC = TFC + TVC$$

Average Cost – Ratio of total cost & output.

$$AC = \frac{TC}{Q}$$

$$AFC = \frac{TFC}{Q} \qquad AVC = \frac{TVC}{Q}$$

Marginal Cost – Change in total cost from a unit change in output.

$$MC = TC_{n+1} - TC_n.$$

Profit – difference between total revenue & total cost.

$$\text{Profit} = \text{Total Revenue} - \text{Total Cost}.$$

MAXIMUM PROFIT.

Maximum Profit is found by differentiating the profit function with respect to the output produced and making the equation equal to zero. After this, second order derivative is done. If second order derivative is less than zero, then the profit is maximum.

→ $\frac{d(\text{Profit})}{dQ} = 0$

→ $\frac{d^2(\text{Profit})}{dQ^2} < 0$

For maximum Profit -

$\frac{d(\text{Profit})}{dQ} = 0$

$\Rightarrow \frac{d(TR-TC)}{dQ} = 0$

$\Rightarrow \frac{d(TR)}{dQ} - \frac{d(TC)}{dQ} = 0$

$\Rightarrow MR - MC = 0$

$\Rightarrow MR = MC.$

BREAK EVEN POINT

The point at which total cost & total revenue are equal.

$$TC = TR$$

$$\therefore \text{Profit} = 0$$

EXERCISE

Ch 04 – Statistical Representation Of Data.

Statistics means the Science of Counting or Science of Averages.

STATISTICS

- STATISTIQUE — French
- STATISTIA — Italian
- STATISTIK — German
- STATUS — Latin

STATISTICS

Singular

- Characteristic
- Collection / Representation of Data
- Scientific Method
- Statistical Methods

Plural

- Qualitative / Quantitative
- Statistical Analysis
- Numerical Data

IMPORTANCE & SCOPE OF STATISTICS

i) Economics

ii) Commerce

iii) Auditing & Accounting

iv) Economic Planning

v) Astronomy

vi) Meteorology

vii) Biology

viii) Mathematics

ix) Research

x) Natural Sciences

xi) Education

xii) Business

LIMITATIONS OF STATISTICS

i) Statistics deals with Aggregates. No significance of Individual data.

ii) Statistics apply only to Quantitative data & not to qualitative data.

iii) Statistical laws are true only on average and not on every single case

iv) Statistics does not reveal the entire story

v) Statistics are liable to be misused

vi) Data should be homogenous & uniform.

DATA

Data is the quantitative information about a particular characteristic that is under study.

Quantitative Characteristic - Variable

Qualitative Characteristic - Attribute

VARIABLE

DISCRETE

When the variable assumes a finite or countable value.

CONTINUOUS

When the variables assumes any value from a given interval.

STEPS FOR DATA ANALYSIS

i) Collection of Data
ii) Classification of Data
iii) Presentation of Data
iv) Analysis of Data.

- **Collection of Data**

DATA

PRIMARY
Data collected for the first time by an investigator.

SECONDARY
Data that is collected by an agency but used by other investigator.

Collection of Primary Data.

i) Interview Method.

- Personal Interview
- Indirect Interview
- Telephonic Interview

ii) Estimates from local sources & correspondence.

iii) Data through Questionnaire.

iv) Questionnaire filled by Enumerators.

Collection of Secondary Data.

Secondary data is collected from any of the following sources –

i) Newspapers & Periodicals
ii) Publications of Trade Associations
iii) Research Papers published by University Departments / UGC.
iv) Official Publication of Central / State Government.
v) Official Publications of Foreign Government / International Organisations.

- **Classification of Data.**

Chronological
Temporal
Time Series

Geographical
Spatial Series

Quantitative
Cardinal

Qualitative
Ordinal

Simple →

Manifold →

Modes of Presentation of Data.

- Textual
- Tabular
- Digrammatic

→ TEXTUAL PRESENTATION

This method of presentation of data means presenting data with the help of numbers & words.

Although this is the easiest way of presenting data, statisticians do not prefer this as it is dull, monotonous and does not help in comparing data.

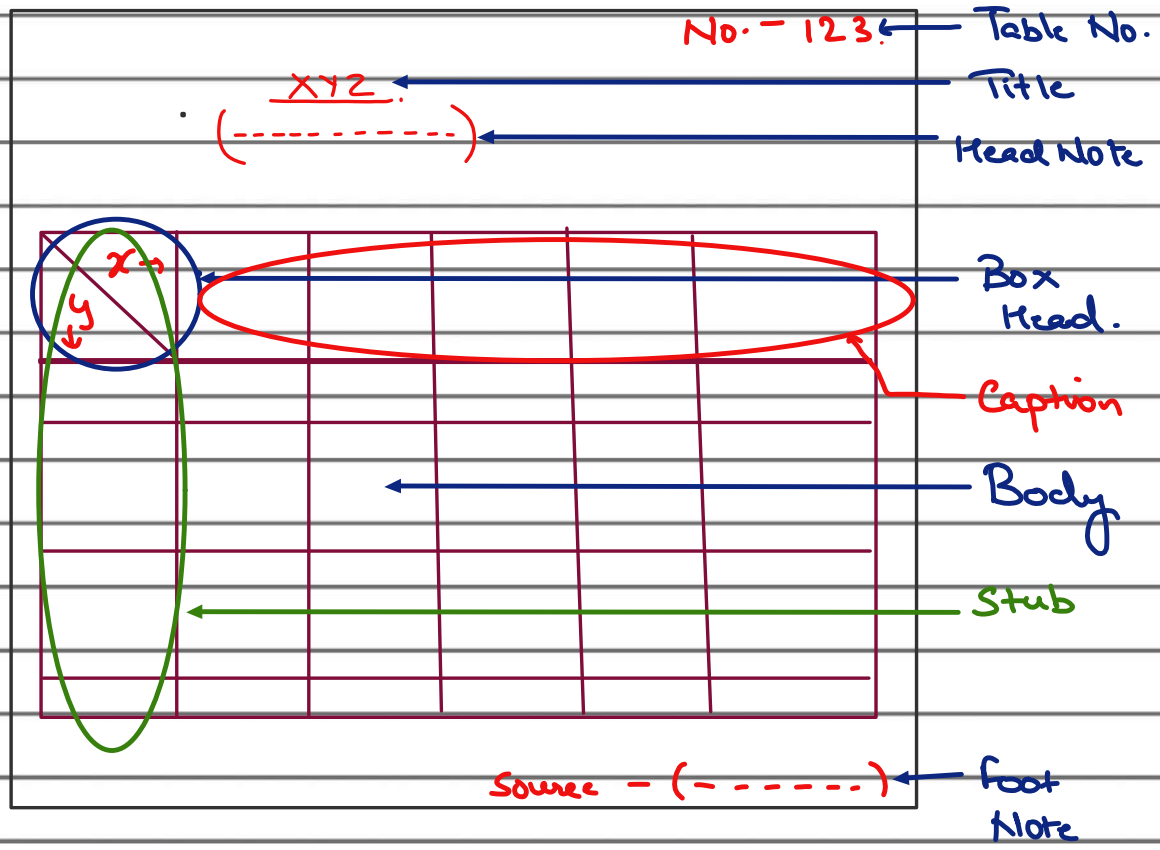
→ TABULAR PRESENTATION

When data is presented in the form of a table, then it is known as tabular presentation of data.

Following are some important terms in tabular presentation -

- Title
- Boxhead
- Stub
- Caption

- Body
- Footnote
- Headnote
- Table No.
- Unit of Measurement
- Date.



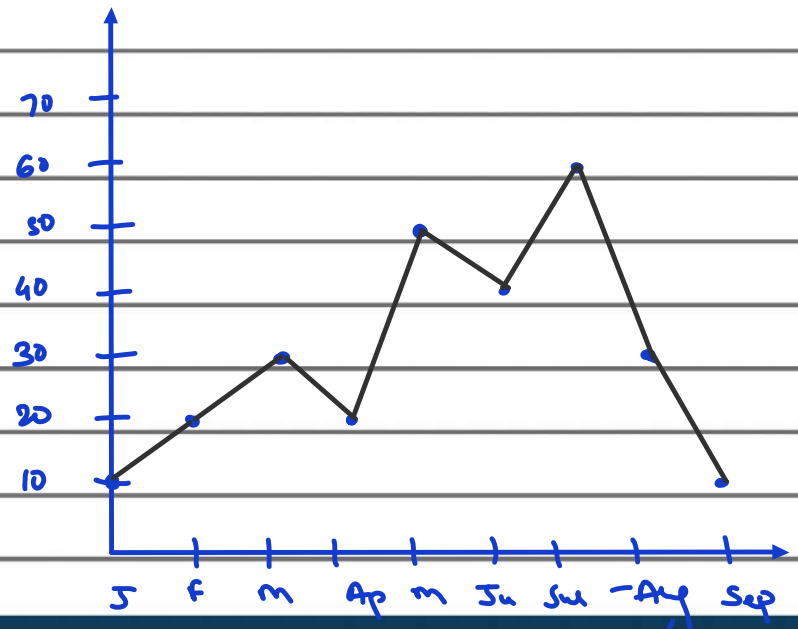
DIAGRAMMATIC REPRESENTATION OF DATA

In this method, different diagrams like lines, bars, charts etc are used to depict data. This type of representation helps in identifying hidden trends in the data.

Diagrammatic Representation
- Line Diagram or HISTORIGRAM
- Bar Diagram
- Pie Chart

* **LINE GRAPH OR HISTORIGRAM**

Line graphs / Line Charts / Historigram are used to represent quantitative data over a specified time interval. All the data points here are connected by a line.



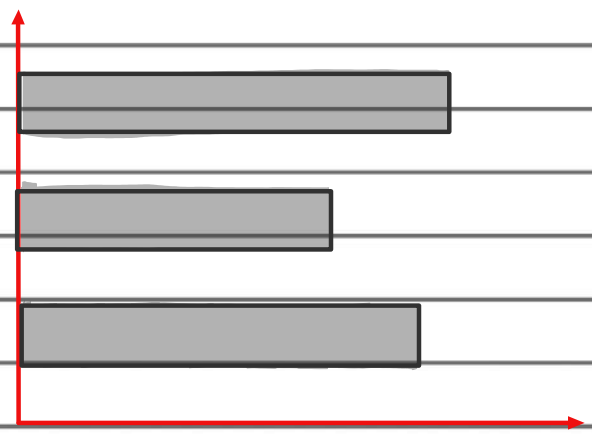
* BAR DIAGRAM OR BAR GRAPH

Bar graphs use horizontal / Vertical bars to represent data such that the length of the bar represents the corresponding data.

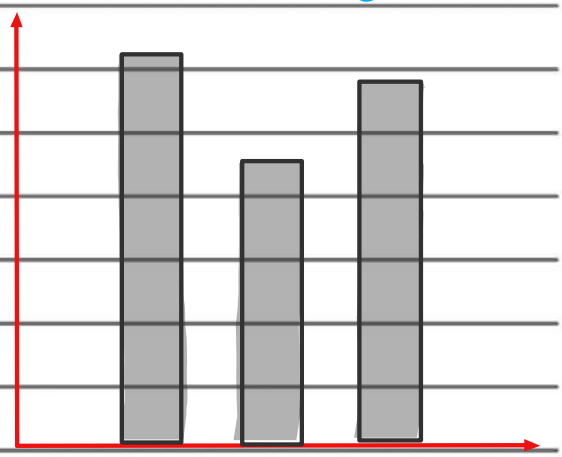
Following are the 4 most commonly used ways of making Bar Graphs -

i) SIMPLE BAR GRAPH

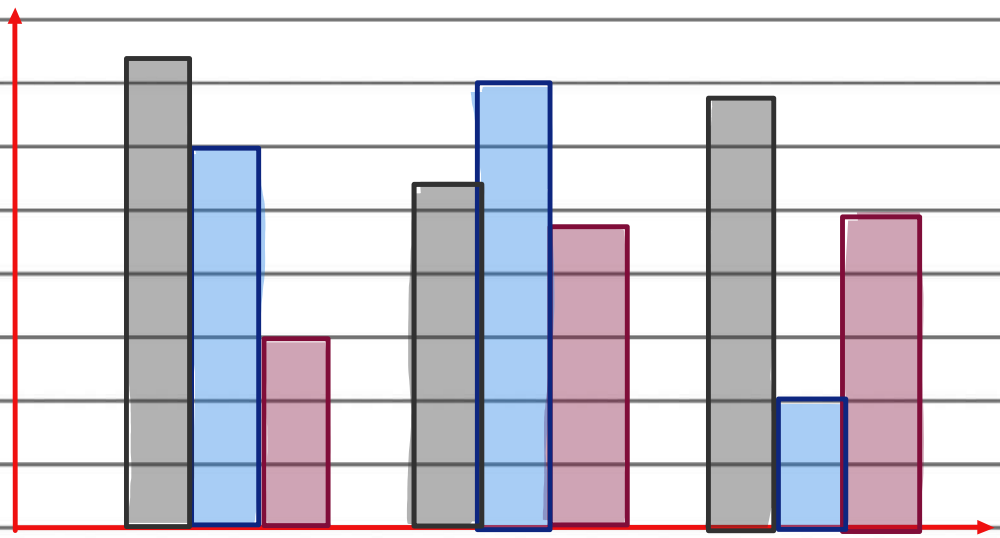
Horizontal Bar Diagram



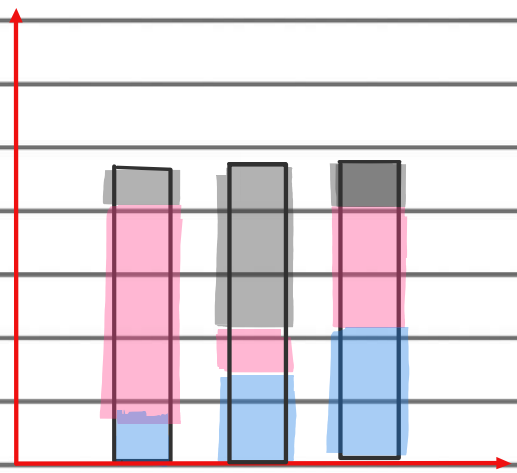
Vertical Bar Diagram



ii) **Multiple / Grouped Bar Diagram** - Multiple attributes of the variable are compared using different coloured bars for each attribute.



iii) **Component / Sub Divided Bar Diagram** - This bar diagram, the aggregate is divided into different components with each component being shown in a different colour.



Apart from these other types of less commonly used Bar Graphs are -

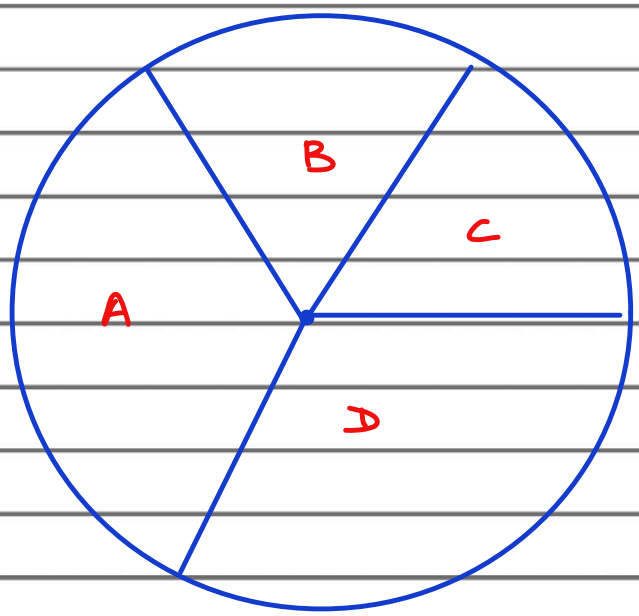
i) Percentage subdivided Bar Graph

ii) Deviation or Bilateral Bar Graph

iii) Broken Bars.

* **PIE CHART**

Here, data is represented in a circular form with the entire circle representing a whole and each part representing a part of the whole.



EXERCISE

FREQUENCY DISTRIBUTION

Frequency means the number of times an observation occurs in any data set.
A Frequency Distribution is a representation that shows the number of time every observation occurs in a data set. It can be in the form of a table, diagram or even charts.

Frequency Distribution Table

Grouped

↓

When distribution is done for continuous variable, a grouped frequency table is formed.

↓

Different classes are formed to show frequency distribution

Ungrouped

↓

When distribution is done for discrete variables, an ungrouped frequency table is formed.

↓

No classes are formed here.

Class	Frequency
0 – 10	
10 – 20	
20 – 30	

Observation	Frequency
2	
5	
7	

Grouped Frequency Table

Overlapping

↓

The upper class limit of the 1st class becomes the lower class limit of the 2nd class & so on.

↓

Class limits & Class Boundaries are the same.

Class	Frequency
0 - 10	
10 - 20	
20 - 30	

Non-Overlapping

↓

There is a difference of the upper class limit of 1st class & lower class limit of the 2nd class & so on.

↓

Class limits & Class Boundaries are different.

Class	Frequency
0 - 10	
11 - 20	
21 - 30	

Important Terms

i) Class limit → The minimum & maximum values of any given class.

ii) Class Boundary → The actual class limit of a class interval in a non-overlapping grouped frequency table.

$$LCB = LCL - \frac{D}{2} \qquad UCB = UCL + \frac{D}{2}$$

where

D = Difference between LCL of 1st Class & UCL of the subsequent class.

iii) Mid Point / Mid Value / Class Mark → The middle value of any class. It is the average of the class limits / boundaries of the respective class.

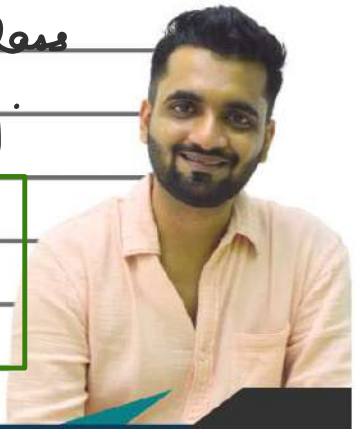
iv) Width / Size / Class Interval → The difference between the class limits or boundaries of any class.

v) Frequency Density → Ratio of the frequency of the class to its class length.

$$\text{Freq. Density} = \frac{\text{Frequency}}{\text{Width}}$$

vi) Relative Frequency → Frequency of a class interval to the total frequency.

$$\text{Rel. Frequency} = \frac{\text{Freq. of the class}}{\text{Total Frequency}}$$



vii) **Percentage Frequency** → Relative frequency expressed as a percentage.

$$\boxed{\% \text{ Frequency} = \text{Rel. Freq.} \times 100.}$$

viii) **Cumulative Frequency** → The type of distribution in which frequencies of all the previous classes are added and shown as the frequency above or below a certain observation.

Less than Cumulative Frequency
↓
Adding the frequencies of all the previous classes along with the frequency of the respective class.

More than Cumulative Frequency.
↓
Subtracting the frequencies of all the previous classes from the total frequency.

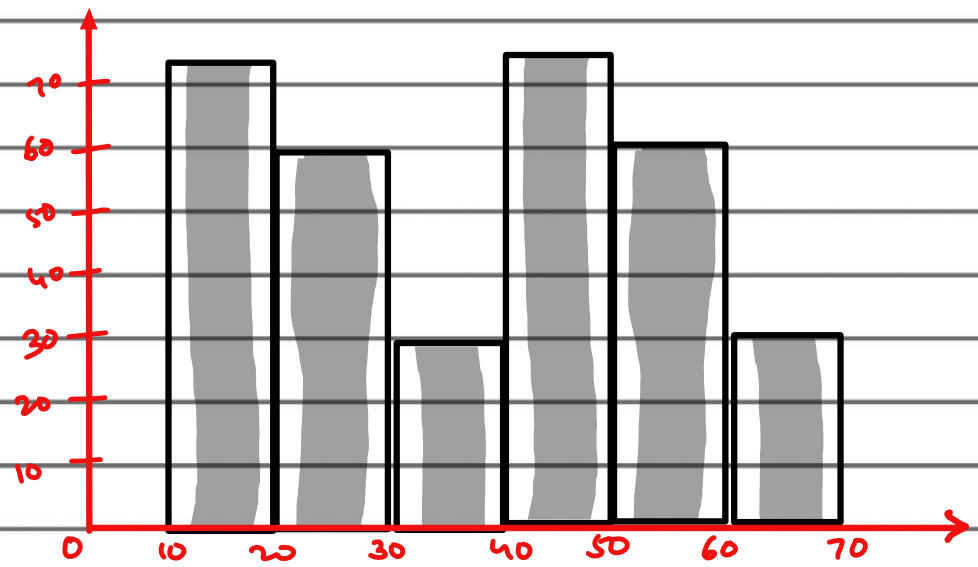
EXERCISE

GRAPHICAL REPRESENTATION OF FREQUENCY DISTRIBUTION

- Histogram
- Frequency Polygon
- Frequency Curve
- Ogives / Cumulative Frequency Curve.

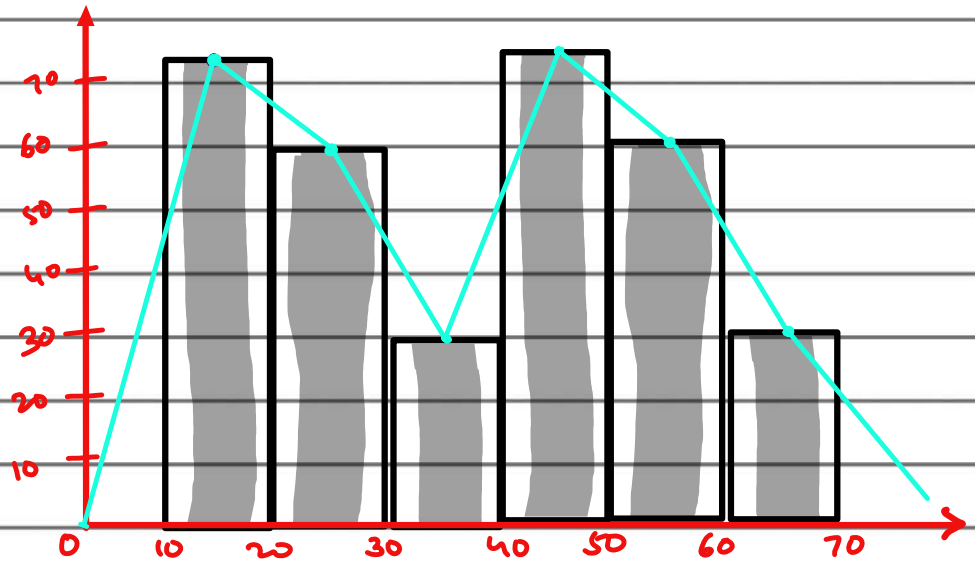
HISTOGRAM

In a histogram, rectangular bars are used to show the frequency of the given classes. Here, class limits are converted into class boundaries and rectangular bars are made corresponding to the classes to show the frequency.



FREQUENCY POLYGON

When the midpoints of all the bars of a histogram are joined using a line, a frequency polygon is formed.

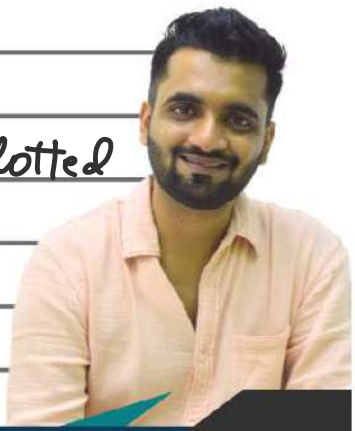


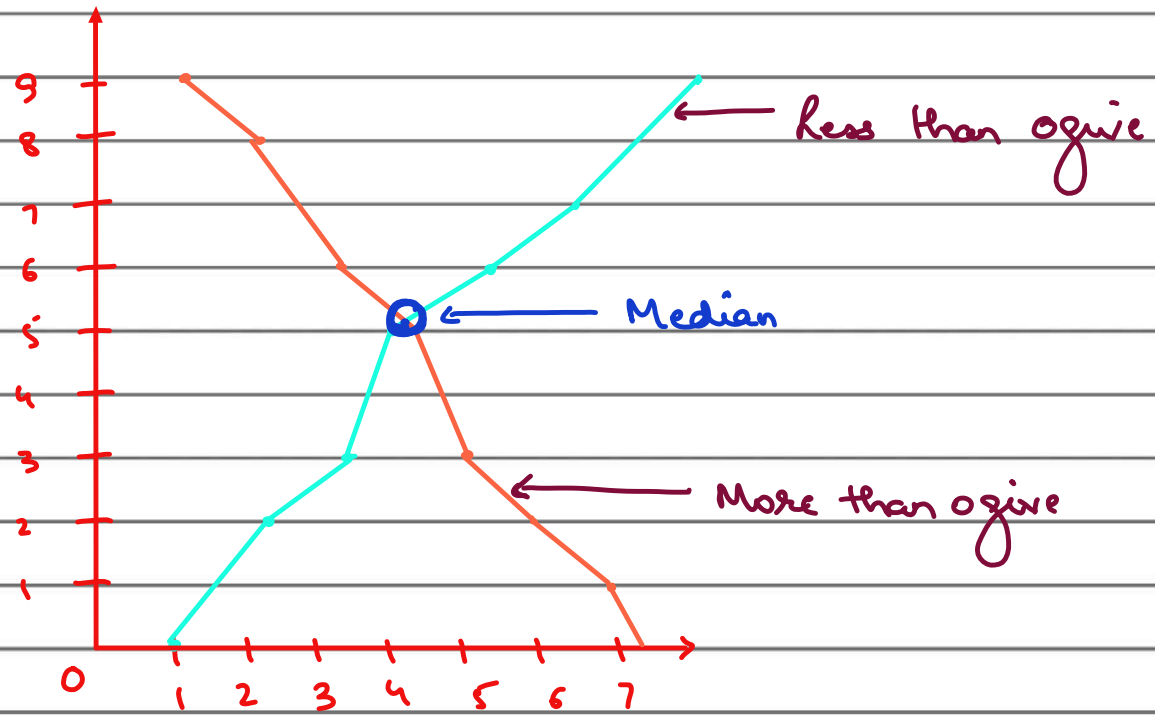
OGIVES

Ogives are used to estimate how many numbers lie above or below a given value in a data. To create an ogive –

→ Less than Cumulative frequency is plotted on the x-axis

→ More than Cumulative frequency is plotted on the y-axis.





Frequency Curve

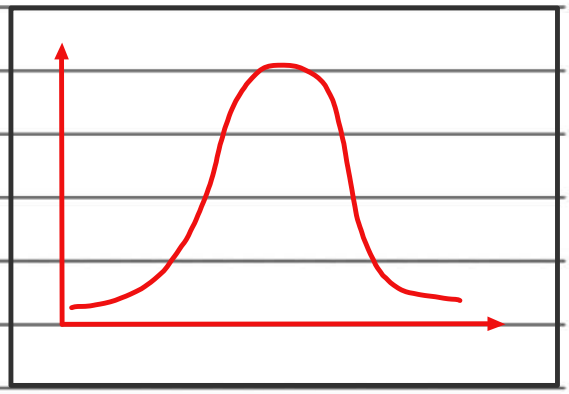
A frequency curve is obtained by drawing a smooth free hand curve through the midpoints of the bars of the histogram.

Frequency Curve

- Bell Shaped
- U-Shaped
- J-Shaped
- Mixed Curve.

BELL SHAPED

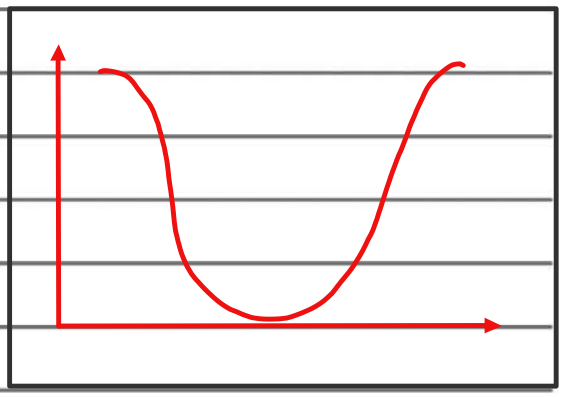
Starts from low value, gradually reaches the maximum value, then gradually decreases to the lowest value.



Eg. - distribution of height, weight, marks etc.

U - SHAPED

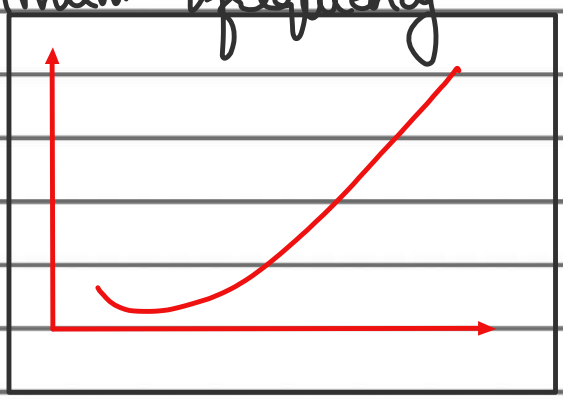
Frequency is minimum at the central part and reaches the maximum at the two extremes.



Eg. - No. of commuters in metro from morning to evening.

J - SHAPED

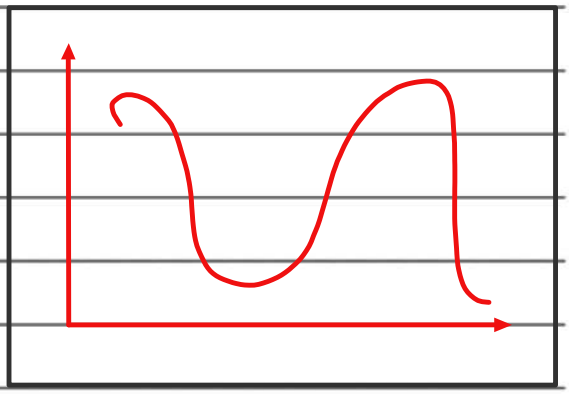
Starts with minimum frequency but gradually reaches to maximum frequency at the other extreme.



Eg. - Commuters in metro from 6am to 10am.

MIXED CURVE

A combination of all of the above without any fixed pattern.



CH 05 - CENTRAL TENDENCY & DISPERSION

MEAN, MEDIAN, MODE

Central tendency of any set of data is a statistical measure that represents its central or typical value.

For a central tendency to be ideal, it should -

i) be properly & rigidly defined
ii) be easy to comprehend
iii) be simple to compute
iv) be based on all the observations
v) be capable of further algebraic treatment
vi) be least affected by the presence of extreme observations.

CENTRAL TENDENCY

- Median
- Mean
  - Arithmetic Mean
  - Geometric Mean
  - Harmonic Mean.
- Mode

ARITHMETIC MEAN (AM)

In a set of data where variable x assumes values x_1, x_2, x_3 etc, the AM is denoted by $\bar{x}$.

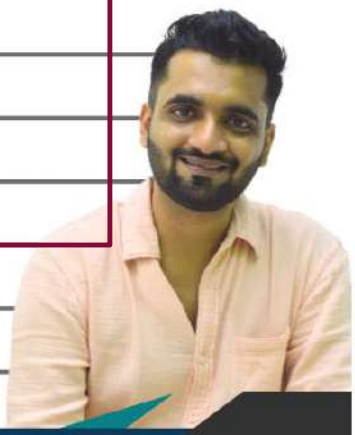
$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 \dots + x_n}{n}$$

$$= \frac{\Sigma x}{n}$$

In case each variable $x_1, x_2, x_3 \dots x_n$ are given with their respective frequencies $f_1, f_2, f_3 \dots f_n$, then

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_3 \dots + x_n f_n}{f_1 + f_2 + f_3 \dots f_n}$$

$$\bar{x} = \frac{\Sigma x_n f_n}{\Sigma f_n}$$



In case classes and their respective frequencies are given, then,

$$\boxed{\bar{x} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_3 \cdots\cdots x_n f_n}{f_1 + f_2 + f_3 \cdots\cdots f_n}}$$

where,

$x_1, x_2, x_3 \ldots$ = Class Mark = $\frac{LCB + UCB}{2}$.

Combined Mean

If in two Sets A & B,

Set A	Set B
n_1 = no. of observations	n_2 = no. of observations
x_1 = Am	x_2 = Am

$$\boxed{\text{Combined Mean } (\bar{x}) = \frac{n_1 \times x_1 + n_2 \times x_2}{n_1 + n_2}}$$

WEIGHTED MEAN

To calculate the weighted mean -

$$\boxed{\bar{X} = \frac{w_1x_1 + w_2x_2 + w_3x_3 - - - - - + w_nx_n}{\sum w_n.}}$$

PROPERTIES OF ARITHMETIC MEAN

i) If all the observations of a dataset are the same (constant - a), then AM of such dataset is also a.

ii) The Algebraic sum of deviations of all the observations from the arithmetic mean is 0.

iii) If every observation of the dataset is increased / decreased by the same number (constant a), then the AM will also get increased / decreased by the same constant.

iv) The Standard error of AM is less than all other measures of Central Tendency

EXERCISE

Geometric Mean (GM).

For Discrete Series -

$$GM = \sqrt[n]{x_1 \times x_2 \times x_3 \ldots\ldots \times x_n}$$

$$= (x_1 \times x_2 \times x_3 \ldots\ldots x_n)^{1/n}.$$

For Continuous Series -

$$GM = \left(x_1^{f_1} \times x_2^{f_2} \times x_3^{f_3} \ldots\ldots x_n^{f_n}\right)^{1/\Sigma f}$$

For Combined GM -

$$\text{Combined } GM = \sqrt[n_1+n_2]{GM_1^{n_1} \times GM_2^{n_2}}$$

$$= \left(GM_1^{n_1} \times GM_2^{n_2}\right)^{1/n_1+n_2}$$

For Weighted GM -

$$\text{Weighted } GM = \sqrt[\Sigma w]{x_1^{w_1} \times x_2^{w_2} \ldots \times x_n^{w_n}}$$

$$= \left(x_1^{w_1} \times x_2^{w_2} \ldots\ldots x_n^{w_n}\right)^{1/\Sigma w}$$

EXERCISE

HARMONIC MEAN (HM)

For Discrete Series -

$$HM = \frac{n}{1/x_1 + 1/x_2 + 1/x_3 \cdots\cdots 1/x_n}$$

If frequencies are also given,

$$HM = \frac{\Sigma f}{f^1/x_1 + f^2/x_2 + f^3/x_3 \cdots\cdots f^n/x_n}$$

For Weighted HM

$$HM = \frac{\Sigma w}{w_1/x_1 + w_2/x_2 + w_3/x_3 \cdots w_n/x_n}$$

Relationship b/w AM, GM & HM

i) For any set of data,

$$AM \geq GM \geq HM$$

ii) If all the values in the dataset are equal,

$$AM = GM = HM.$$

iii) If all the values in the dataset are distinct,

$$AM > GM > HM.$$

$$\boxed{GM^2 = AM \times HM.}$$

EXERCISE

MEDIAN

Median is the middle value in any distribution when it is arranged in an ascending or descending order.

Median for Discrete Series

When the number of observation is

ODD

$$M_e = \left(\frac{n+1}{2}\right)^{th} \text{ Observation}$$

EVEN

$$M_e = \frac{\Sigma 2 \text{ Middle Obs}}{2}$$

Median for Continuous Series

$$M_e = LCB + \left[\frac{\frac{1}{2}n - n_l}{n_u - n_l}\right] \times C$$

Steps to find the Median

i) Find LCB for each class & create

a less than type cumulative frequency table.

ii) Find $\frac{1}{2}N$ (where n = total frequency)

iii) Class containing $\frac{1}{2}n$ is the Median class.

iv) Find -

N_l = CF smaller than $\frac{1}{2}N$

N_u = CF greater than $\frac{1}{2}N$

LCB = Class Boundary of Class having (N_l).

C = Class Length.

EXERCISE

QUARTILES / DECILES / PERCENTILES

QUARTILES → Quartiles are observation that divide any data set in 4 equal parts. with each part containing 25% of the observations. There are three quartiles i.e; Q_1, Q_2 & Q_3. Q_2 is the same as Median.

DECILES → Deciles are observation that divide any data set in 10 equal parts. with each part containing 10% of the observations. There are nine deciles - D_1, D_2, D_3 ..... D_9. D_5 is the same as Median.

PERCENTILES → Percentiles are observation that divide any data set in 100 equal parts. with each part containing 01% of the observations. There are ninety nine percentiles - P_1 P_2 .... P_{99}. P_{50} is the same as Median.

QUARTILES

DISCRETE

$$Q_i = \frac{i(N+1)^{th}}{4}$$

CONTINUOUS

$$Q_i = LCB + \left[\frac{\frac{i}{4}N - N_L}{N_U - N_L}\right] \times C$$

when,

N = Total Observation

N = Total frequency

STEPS

i) Find $\frac{i}{4}N$

N_L = CF smaller than $\frac{1}{4}N$

N_U = CF greater than $\frac{1}{4}N$

LCB = Class Boundary of Class having (N_L).

C = Class Length.

ii) Put the above values in the given formula.

DECILES

DISCRETE

$$D_i = i\,\frac{(N+1)^{th}}{10}$$

CONTINUOUS

$$D_i = LCB + \left[\frac{\frac{i}{10}N - N_L}{N_u - N_l}\right] \times C$$

when,

N = Total Observation

N = Total frequency

STEPS

i) find $\frac{i}{10}N$

N_l = CF smaller than $\frac{1}{10}N$

N_u = CF greater than $\frac{1}{10}N$

LCB = Class Boundary of Class having (N_l).

C = Class Length.

ii) Put the above values in the given formula.

PERCENTILES

DISCRETE

$$P_i = \frac{i(N+1)^{th}}{100}$$

CONTINUOUS

$$P_i = LCB + \left[\frac{\frac{i}{100}N - N_L}{N_u - N_L}\right] \times C$$

when,

N = Total Observation

N = Total frequency

STEPS

i) find $\frac{i}{100}N$

N_L = CF smaller than $\frac{i}{100}N$

N_u = CF greater than $\frac{i}{100}N$

LCB = Class Boundary of Class having (N_L).

C = Class Length.

ii) Put the above values in the given formula.

EXERCISE

MODE

For a given set of observations, Mode is that observation which occurs the maximum number of times.

In a Discrete Series, Mode is calculated by mere Inspection. The Observation having the highest frequency is the mode.

For Continuous Series

$$M_o = LCB + \left[\frac{f_0 - f_{-1}}{(2f_0) - (f_1) - (f_{-1})} \times C\right]$$

where,

f_0 = frequency of modal class

f_1 = frequency of class after modal class

f_{-1} = frequency of class before modal class

C = Class length.

* Modal Class → Class with the highest frequency.

RELATION b/w MEAN, MEDIAN, MODE

- Mean − Mode = 3 (Mean − Median)
- Mean − Mode = 3Mean − 3Median
- 3 Median − Mode = 2 Mean
- 3 Median = 2 Mean + Mode.

EXERCISE

Range, Mean Deviation, Quartile Deviation

Dispersion is the measurement of scatterness or spread of data points. It measures how much data points deviate from the central tendency.

Measures of Dispersion

Absolute Measures

→ Depends on the unit

Eg. - Range
Mean Deviation
Standard Deviation
Quartile Deviation

Relative Measure

→ Are Unit free
→ Used for comparing two or more distribution

Eg. - Coefficient of Range
Coefficient of MD
Coefficient of SD
Coefficient of Variation

Graphical Measure of Dispersion

↓

Lorenz Curve

RANGE (R)

DISCRETE

$$R = \text{Largest Obs.} - \text{Smallest Obs.}$$

CONTINUOUS

$$R = \text{UCB (Last Class)} - \text{LCB (First Class.)}$$

$$\text{Coefficient of Range} = \frac{\text{Range}}{\text{Largest Obs} + \text{Smallest Obs}} \times 100$$

$$= \frac{L - S}{L + S} \times 100$$

* NOTE

Range remains UNCHANGED due to change in ORIGIN.

Range CHANGES in the same ratio due to change in SCALE.

Quasi Range (QR)

$$QR = 2^{nd} \text{ highest observation} - 2^{nd} \text{ lowest observation}$$

Interquartile Range (IQR)

$$IQR = Q_3 - Q_1$$

EXERCISE

Quartile Deviation (QD)

It is also called semi inter quartile range. It measures the spread of data around the median.

$$QD = \frac{Q_3 - Q_1}{2}.$$

Coefficient of QD

$$\text{Coefficient of QD} = \frac{QD}{\text{Median}} \times 100.$$

$$= \frac{Q_3 - Q_1/2}{Q_3 + Q_1/2} \times 100$$

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100.$$

EXERCISE

MEAN DEVIATION (MD)

Also known as Average Deviation, MD is a arithmetic mean of the absolute deviation of the observations from an appropriate measure of central tendency.

Mean Deviation (MD)

DISCRETE

$$MD = \frac{\Sigma |x - \bar{x}|}{n}$$

x = observation
$\bar{x}$ = AM of all observation
n = No. of total observation

CONTINUOUS

$$MD = \frac{\Sigma f_i |x_i - \bar{x}|}{\Sigma f}$$

$\bar{x}$ = AM = $\frac{\Sigma f_i x_i}{n}$
x_i = Observation
f_i = Respective Freq.
n = No. of total Observations.

Steps for Calculation of MD

i) Find AM $\left(\bar{x} = \frac{\Sigma f_i x_i}{n}\right)$

ii) Make the following table

Class Interval	Class Mark x	Frequency f	$x-\bar{x}$	$f\lvert x-\bar{x}\rvert$
In case of Continuous Series				

* NOTE

MD takes its minimum value when taken from MEDIAN.

$$\boxed{\text{Coefficient of Mean Deviation} = \frac{MD}{\text{Mean}} \times 100.}$$

$$\boxed{\text{Coefficient of MD about Median} = \frac{MD}{\text{Median}} \times 100.}$$

EXERCISE

STANDARD DEVIATION (δ)

SD is the square root of the arithmetic mean of the squares of deviation of observation from their AM.

For Discrete Series -

$$\delta = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\delta = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

where,

$\bar{x}$ = mean

n = no. of observations

x = Individual observation

For Continuous Series,

$$\boxed{\delta = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f}}}$$

$$\boxed{\delta = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - (\bar{x})^2}}$$

Steps for Calculation of SD (δ)

i) Find AM $\left(\frac{\Sigma f_i x_i}{\Sigma f} = \bar{x}\right)$

ii) Make the following table

Class Interval	Class Mark x	Frequency f	$x - \bar{x}$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
Case of Continuous Series					

Properties of SD

i) SD of 1st 'n' natural numbers -

$$\boxed{\delta = \sqrt{\frac{n^2-1}{12}}}$$

ii) If all the variable in a particular data set is equal, then δ is equal to 0.

iii) SD is independent of change in origin but NOT independent of change in SCALE.

iv) For any dataset, SD $\geq$ MD.

v) For any dataset, $\Sigma(x-\bar{x})^2$ is always the least.

EXERCISE

COMBINED STANDARD DEVIATION

When two sets of data are given such that

SET 1	SET 2
n_1 = No. of obs	n_2 = no. of obs
$\bar{x}_1$ = AM	$\bar{x}_2$ = AM
S_1 = S.D	S_2 = S.D

$$\boxed{\text{Combined Deviation } (\delta) = \sqrt{\frac{n_1(S_1^2+d_1^2)+n_2(S_2^2+d_2^2)}{n_1+n_2}}}$$

where,

$$d_1 = \bar{x}_1 - \bar{x}$$

$$d_2 = \bar{x}_2 - \bar{x}$$

$$\bar{x} = \text{Combined Mean} = \frac{x_1 n_1 + x_2 n_2}{n_1+n_2}.$$

EXERCISE

VARIANCE (δ^2)

The square of standard deviation is known as variance.

For Discrete Series -

$$\boxed{\delta^2 = \frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\boxed{\delta^2 = \frac{\Sigma x^2}{n} - \bar{x}^2}$$

For Continuous Series,

$$\boxed{\delta^2 = \frac{\Sigma f_i(x_i-\bar{x})^2}{\Sigma f}}$$

$$\boxed{\delta^2 = \frac{\Sigma f x^2}{\Sigma f} - (\bar{x})^2}$$

EXERCISE

COEFFICIENT OF VARIATION (CoV)

$$\boxed{CoV = \frac{SD}{AM} \times 100}$$

* Higher the CoV, higher will be variation of observations in the data.

Relationship b/w different measures of Dispersion -

$$\boxed{\begin{aligned} QD &= \frac{2}{3} SD \\ MD &= \frac{4}{5} SD \end{aligned}}$$

Further,

i) $\bar{x} \pm QD$ covers 50% of terms.

ii) $\bar{x} \pm MD$ covers 57.51% of terms.

iii) $\bar{x} \pm SD$ covers 68.27% of items.

iv) $\bar{x} \pm 2SD$ covers 95.45% of terms

v) $\bar{x} \pm 3SD$ covers 99.73% of items.

EXERCISE

CH 06 - CORRELATION REGRESSION

A. CORRELATION

Correlation is a statistical measure that extent to which two or more variable fluctuate in relation to each other.

A Positive Correlation means that when one variable increases, the other variable also increases.

A Negative Correlation means that when one variable increases, the other variable decreases.

Zero Correlation means there is no relation between the two variables.

CORRELATION

- POSITIVE
  - Perfect +ve → $x = 1$
  - +ve → $0 < x < 1$
- ZERO → $x = 0$
- NEGATIVE
  - -ve → $-1 < x < 0$
  - Perfect -ve → -1

SPURIOUS CORRELATION

SIMPLE & MULTIPLE CORRELATION

Simple Correlation is a situation in which only two variables exist and a correlation study is done in b/w the said two variables only.

Multiple Correlation mean the correlation b/w one variable and all other variables taken together in situations where more than two variables exist.

PARTIAL & TOTAL CORRELATION

When there are more than two variables and the relationship b/w any two of them is studied assuming all the other

variables being constant, then it is known as partial correlation.

When the correlation among the variables is worked out taking all at a time, it is known as total correlation.

MEASURES OF CORRELATION

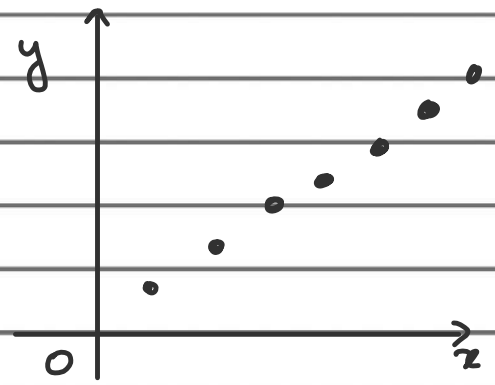
i) Scatter Diagram
ii) Karl Pearson's Product Moment Correlation Coefficient
iii) Spearman's Rank Correlation Coefficient
iv) Co-efficient of Concurrent Deviation.

SCATTER DIAGRAM

In a Scatter Diagram, data is plotted on a graph with one variable on the x-axis and the other variable on the y-axis. The way in which the points fall on the graph give us an idea of the correlation between the two variables.

TYPES OF GRAPHS

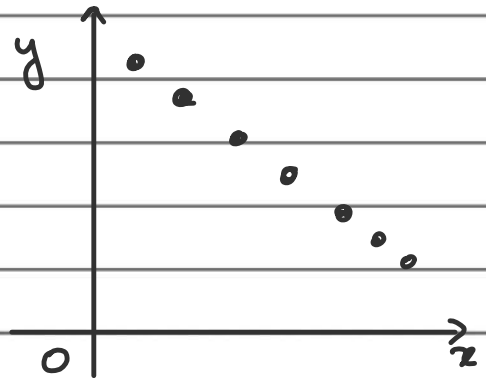
i)



Perfect Positive Correlation

$r = 1$

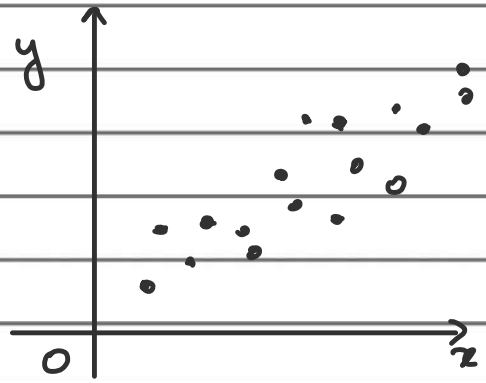
ii)



Perfect Negative Correlation

$r = -1$

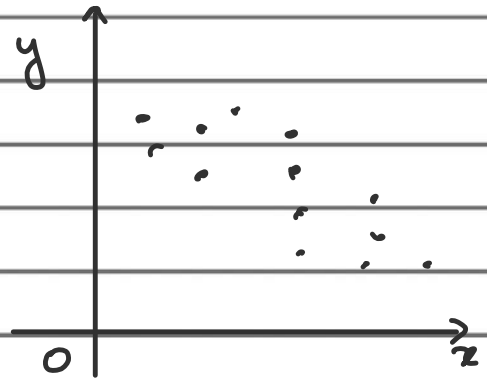
iii)



Positive Correlation

$r = 0 < r < 1.$

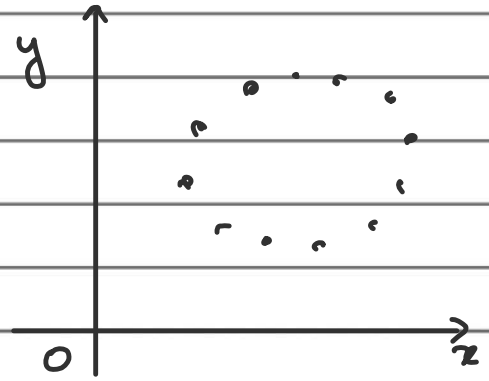
iv)



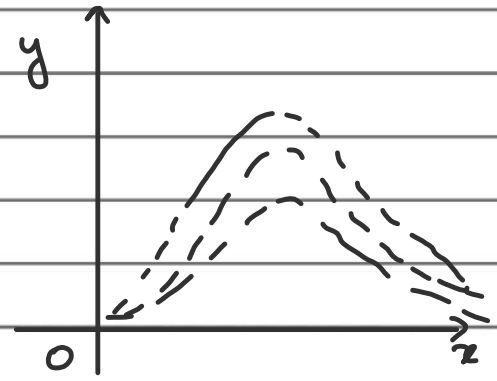
Negative Correlation

$r = -1 < r < 0$

v)



$r = 0$



$r = 0$

No Correlation.

NOTE

Scatter diagrams help to understand only the nature of correlation between two variables and not the exact correlation between the variables.

KARL PEARSON'S PRODUCT MOMENT CORRELATION COEFFICIENT

This method is often called the BEST METHOD for finding the correlation coefficient. (r).
The properties of product moment correlation coefficient are -

i) It is unit free.
ii) It DOES NOT CHANGE with change in Scale or Origin.
iii) r always lies between -1 and 1, including both.

$$r = \frac{Cov(x,y)}{\delta_x \times \delta_y} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$Cov(x,y) = \frac{\Sigma xy}{n} - \bar{x}\bar{y}$$

$$\delta_x = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$\delta_y = \sqrt{\frac{\Sigma y^2}{n} - \bar{y}^2}$$

x = 1st Variable
y = 2nd Variable
$\bar{x}$ = Am of x
$\bar{y}$ = Am of y
n = Total no. of Observation

x	y	xy	x^2	y^2
Σx	Σy	Σxy	Σx^2	Σy^2

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\bar{y} = \frac{\Sigma y}{n}$$

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
				Σ	Σ	Σ

PROBABLE ERROR

$$PE = 0.6745 \frac{(1-r^2)}{\sqrt{n}}$$

r = coefficient of correlation
n = no. of pairs of observations

PROPERTIES of PE

i) If $r < PE$, then there is NO CORRELATION.

ii) If $r > PE$, then there is SIGNIFICANT CORRELATION.

iii) If $r \not> 6$ times PE, then the Correlation is not much significant.

iv) If PE is small, then correlation is definitely existing at $r > 0.5$

v) If $r > 0.3$, PE is very small.

EXERCISE

Spearman's Rank Correlation Coefficient

Rank Correlation (r_R) is used to find correlation between variables that deal with qualitative factors. This method is also applied to find level of agreement / disagreement between two judges.

$$r_R = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

where,

d = difference in ranks of x & y
$= r_x - r_y$.

n = Total no. of observations.

x	y	Rank x	Rank y	d (Rank$_x$ - Rank$_y$)	d^2
					Σd^2

Exercise

In case of tied ranks, following steps are to be followed -

i) Ranks of all the tied observations will be an average of their rank wise positions.

Eg. -	21	20	19	23	20	27	23	23
Rank -	5	6.5	8	3	6.5	1	3	3

ii) Make a table as is normally made to find r_R.

iii) Find $\frac{\Sigma t^3 - t}{12}$ for all the tied observation as follows -

Eg. - Tied Obs.	No. of ties	$\frac{t^3-t}{12}$
20	2	0.5
23	3	2
		Σ 2.5

$\therefore \frac{\Sigma t^3 - t}{12} = 2.5$

iv) Put the value in the following equation -

$$r_R = 1 - \frac{6\left[\Sigma d^2 + \frac{\Sigma t^3 - t}{12}\right]}{n(n^2-1)}$$

EXERCISE

Coefficient of Concurrent Deviations

This method is used to measure the degree of simultaneous variation or deviation in two variables

$$\boxed{r_c = \pm\sqrt{\pm\frac{2c-m}{m}}}$$

where

c = No. of concurrent deviations
ie; no. of x & y moving in the same direction.

$m = n-1$.

NOTE

If $\frac{2c-m}{m} = -ve$

$$r_c = -\sqrt{-\frac{2c-m}{m}}$$

If $\frac{2c-m}{m} = +ve$

$$r_c = \sqrt{\frac{2c-m}{m}}$$

x	y	deviation x_1 to x_2	deviation y_1 to y_2	Whether deviation in same direction	
2	7				
		Up	Down	X	
3	6				
		Up	Up	✓	
5	7				$c = 2.$
		Down	Up	X	
4	8				
		Down	Down	✓	
3	7				

EXERCISE

B. REGRESSION ANALYSIS

Regression Analysis is done to estimate the relationship between one dependant variable and another independant variable. It helps in understanding how one variable affects the other variable.

TYPES OF REGRESSION ANALYSIS

SIMPLE

One dependant
One Independant variable

MULTIPLE

One dependant
Multiple Independant variables.

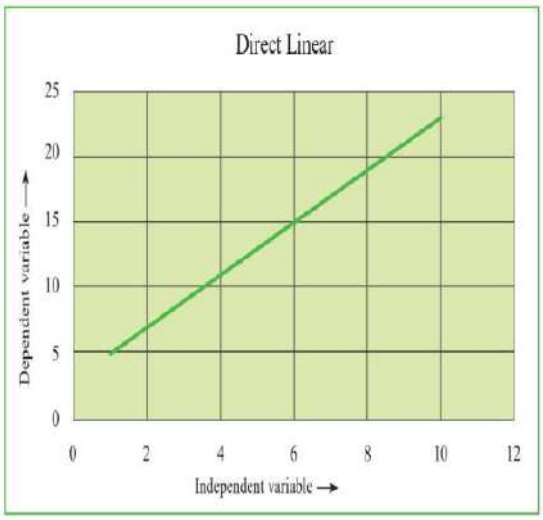
PARTIAL

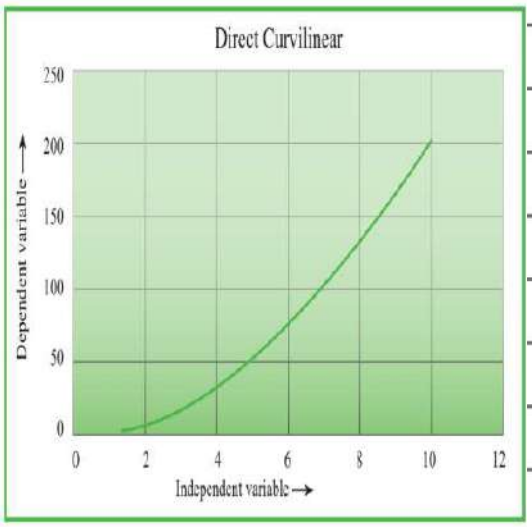
Multiple variables exist but relationship b/w only two variables is analyzed at one time.

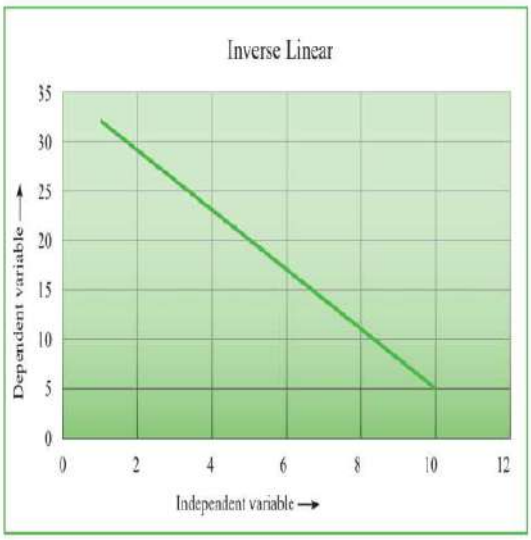
On the basis of type of relation

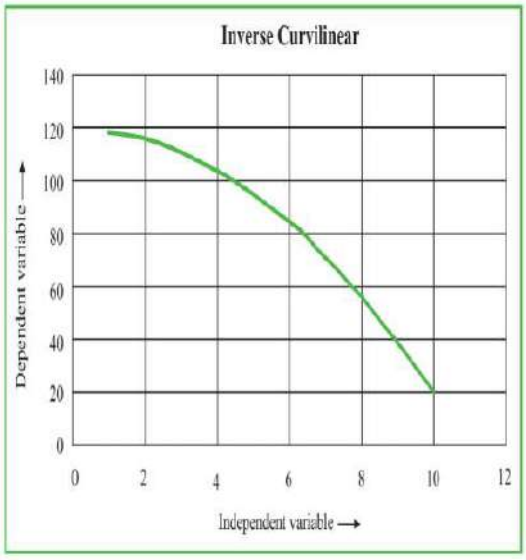
LINEAR

NON LINEAR









Regression lines are found by using the METHOD OF LEAST SQUARES.

In case of two variables x and y, if variable y is dependant on x, then in such cases,

y = dependant variable / Regression / Explained Variable.

x = Independant Variable / Predictor / Explanator.

In cases where y depends on x, then the regression line of y on x is given by -

$$y = a + bx$$

where,

a & b = Regression parameters

b = Regression Co-efficient.

Estimation of x, when y is given -

x = dependant

y = Independant

↓

x on y

Regression line of x on y

$$\boxed{x - \bar{x} = b_{xy}(y - \bar{y})}$$

$$\bar{x} = \frac{\Sigma x}{n} \qquad \bar{y} = \frac{\Sigma y}{n}$$

$$\boxed{b_{xy} = \frac{n \times \Sigma xy - \Sigma x \times \Sigma y}{n\,\Sigma y^2 - (\Sigma y)^2}}$$

$$\boxed{b_{xy} = \frac{Cov(x,y)}{Var.\ of\ y} \quad \text{or} \quad \frac{r \times \delta x}{\delta y}}$$

$$\boxed{b_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(y-\bar{y})}}$$

Estimation of y when x is given -

y = dependant
x = Independant
↓
y on x

Regression line of y on x

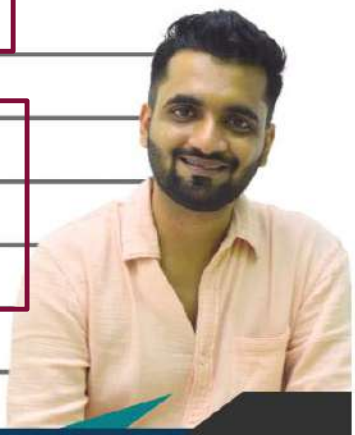
$$\boxed{y - \bar{y} = b_{yx}(x - \bar{x})}$$

$$\bar{y} = \frac{\Sigma y}{n}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\boxed{b_{yx} = \frac{n\Sigma xy - \Sigma x \times \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}}$$

$$\boxed{b_{yx} = \frac{Cov(x,y)}{Var\ of\ x} \quad or \quad \frac{r \times \sigma_y}{\sigma_x}}$$



$$b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

x	y	xy	x^2	y^2
Σx	Σy	Σxy	Σx^2	Σy^2

EXERCISE

PROPERTIES OF REGRESSION LINES

i) Remains UNCHANGED – Change in Origin
CHANGES – Change in Scale.

ii) The two regression lines (x on y & y on x) intersect at a point which gives values for $\bar{x}$ & $\bar{y}$.

iii) Coefficient Correlation is equal to the simple geometric mean of the two regression coefficients between the same two variables x & y

$$r = \pm\sqrt{b_{xy} \times b_{yx}}.$$

iv) If both (b_{xy} & b_{yx}) are negative → r will be negative

If both (b_{xy} & b_{yx}) are positive → r will be positive.

NOTE

b_{xy} & b_{yx} cannot be of different signs.

v) The AM of b_{xy} & b_{yx} is greater than or equal to r.

$$\frac{b_{xy}+b_{yx}}{2} \geq r.$$

Relation b/w Correlation & Regression Coefficients

$$r = \pm\sqrt{b_{xy} \times b_{yx}}.$$

If there are two equations,

$$\boxed{ax + by + c = 0} \quad \hookrightarrow (1)$$

$$\boxed{mx + ny + d = 0} \quad \hookrightarrow (2)$$

Follow the following steps -

i) Assume Eq (1) as Y on x and Eq (2) as x on y.

then,

$$y = \frac{-c}{b} - \frac{ax}{b} \qquad x = \frac{-d}{m} - \frac{ny}{m}$$

$$\therefore b_{yx} = \frac{-a}{b} \qquad b_{xy} = \frac{-n}{m}.$$

ii) Put the values of b_{yx} & b_{xy} in the following equation

$$r = \pm\sqrt{b_{xy} \times b_{yx}}.$$

If r comes to be between -1 & 1, then assumption of equation is right.

If r is not between/equal to -1 or 1, then assumption is to be reversed.

EXERCISE

STANDARD ERROR OF ESTIMATE

$$S_e = \sqrt{\frac{\Sigma Y^2 - a\Sigma Y - b\Sigma XY}{n-2}}$$

NOTE →

i) 68% of points lie within $\pm 1 S_e$

ii) 95.5% of points lie within $\pm 2 \times S_e$

iii) 99.7% of points lie within $\pm 3 \times S_e$.

If $S_e = 0$, all the value of Y will lie exactly on the regression line.

EXERCISE

Ch 07 - PROBABILITY

Probability is a concept that helps in quantifying UNCERTAINITY. It provide tools for analyzing random events.

Probability essentially means the measurement of likelihood of an event. Probability of any event is expressed as a number between 0 and 1. It is shown as $P(\text{Event}) = x$.

Probability of 0 means that the event is impossible and probability of 1 means that the event is certain.

Important Terms

- **Experiment** - Performance of a certain task to achieve a certain result.
- **Random Experiment** - An experiment for which the result is uncertain and is based solely on chance.
- **Events** - Events are the result or outcomes of a certain experiment.

Types Of Events

i) **Simple** - An event is a simple event if it cannot be decomposed into further events.
Eg. - Tossing of a coin

ii) **Composite** - An event that can be decomposed into further events.
Eg. - Getting a head when two coins are tossed

iii) **Mutually Exclusive / Incompatible** - Events that cannot occur simultaneously.
Eg. - Getting a H or T when tossing a coin.

iv) **Exhuastive** - The set of all possible outcomes of an experiment are called exhaustive events. In such cases, at least one of the outcomes must occur on happening of experiment.
Eg - H and T when tossing a coin.

v) **Equally Likely / Mutually Symmetric / Equi-Probable** - Events that have the same probability of happening.
Eg. - H or T when tossing a coin.

CLASSICAL / A PRIORI DEFINITION

As per this definition, the probability of happening of an event (A) is given by -

$$\boxed{P(A) = \frac{n_A}{n}}$$

where,

n_A = no. of favourable events
n = no. of total events.

* NOTE → $n_A \leq n$.

Properties of Classical Definition

i) Applicable when total no. of events is finite.
ii) Used only when outcomes are exclusive or equi probable.
iii) Has limited application where possible events are known in advance.

iv) $0 \leq P_A \leq 1$ → P_A cannot be > 1 or < 0.

v) While probability of occurrence of an event is given by $P(A)$, probability of non-occurrence of the same event is

given by $P(A')$ or $P(A^c)$.

$$P(A) + P(A') = 1$$

$$P(A') = 1 - P(A)$$

$$P(A) = 1 - P(A')$$

vi) The ratio of no. of favourable events to the no. of unfavourable events is known as odds in favour of the event, while vice versa is known as odds against the event.

$$\text{Odds in favour of } A = \frac{n_A}{n - n_A}.$$

$$\text{Odds against } A = \frac{n - n_A}{n_A}$$

EXERCISE

Addition Theory

Theorem 1

For two mutually exclusive events A and B

$$\boxed{P(A \cup B) = P(A) + P(B)}$$

Theorem 2

For any sequence of mutually exclusive events $A_1, A_2, A_3, \ldots A_k$

$$\boxed{P(A_1 \cup A_2 \cup A_3 \ldots \cup A_k) = P(A_1) + P(A_2) \ldots + P(A_k)}$$

Theorem 3

For any 2 events A and B, the probability that any and only one of the events will happen is given by $P(A \cup B)$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

THEOREM 4

For any 3 events A, B and C, the probability that any and only one of them will occur is given by $P(A \cup B \cup C)$

$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) - P(A \cap B \cap C).}$$

EXERCISE

MULTIPLICATION THEORY

When A and B are Independent Events,

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

Conditional Probability for two events A and B can be denoted in two forms

B given A → Probability of B given that A has already occurred. $P\left(\frac{B}{A}\right)$

A given B → Probability of A given that B has already occurred. $P\left(\frac{A}{B}\right)$

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \qquad P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$$

$$P(A \cap B) = P(B) \times P\left(\frac{A}{B}\right).$$

$\therefore$ COMPOUND PROBABILITY $P(A \cap B)$.

Dependent Event

$$P(A \cap B) = P\left[\frac{A}{B}\right] \times P(B)$$

$$P(A \cap B) = P\left[\frac{B}{A}\right] \times P(A)$$

Independent Event

$$P(A \cap B) = P(A) \times P(B).$$

EXERCISE

CH 08 - INDEX NUMBERS

An Index number is a ratio of two or more time periods involved, One of which is the base time period. The value of the base time period acts as a standard point for Comparison.
Eg - NSE, BSE etc.

Index numbers help in measuring changes over time or across locations.

INDEX NUMBERS

- **Simple** → Calculated for a single variable
- **Composite** → Calculated for two or more variables.

TYPES of Index Numbers

i) PRICE INDEX
ii) QUANTITY INDEX
iii) VALUE INDEX
iv) DIFFUSION INDEX
v) OTHERS

METHODS FOR MAKING INDEX NUMBERS

- Simple
  - Aggregative
  - Relative
- Weighted
  - Aggregative
  - Relative.

→ SIMPLE AGGREGATIVE METHOD

The total of commodity prices in a given year are shown as a percentage of total commodity prices of the base year.

$$\text{Price Index } (P_{01}) = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

→ WEIGHTED AGGREGATIVE METHOD

$$P_{01} = \frac{\Sigma P_1 W}{\Sigma P_0 W} \times 100$$

There are four alternatives in this method -

a) Laspeyre's Index Number

$$P_{01} = \frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100$$

(Base Year Quantities are used.)

b) Paasche's Index Number

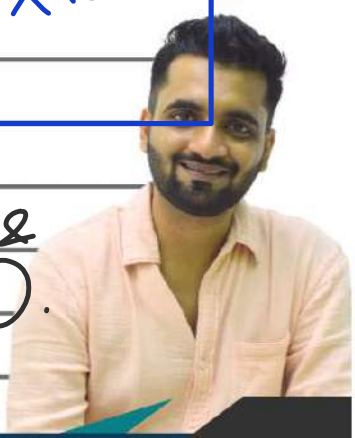
$$P_{01} = \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times 100$$

(Current Year Quantity is used)

iii) Fisher's Index Number

$$P_{01} = \sqrt{L \times P} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times 100$$

(Geometric Mean of Laspeyre's & Paasche's Index Numbers).



iv) Bowley's Method.

$$P_{01} = \frac{L+P}{2} \times 100 = \frac{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} + \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}}{2} \times 100$$

(Arithmetic Mean of Laspeyre's & Paasche's Index Numbers).

EXERCISE

→ **Simple Average of Relatives**

$$P_{01} = \frac{1}{n} = \Sigma \left[\frac{P_1}{P_0} \times 100 \right]$$

$$P_{01} = \text{Antilog} \left[\Sigma \log(P) / n \right]$$

→ **Weighted Average of Relatives**

$$P_{01} = \frac{\Sigma P\omega}{\Sigma \omega} = \frac{\Sigma \left[\left(\frac{P_1}{P_0} \times 100 \right) w \right]}{\Sigma \omega}.$$

Quantity Index Numbers

i) Simple Aggregative $= \frac{\Sigma Q_1}{\Sigma Q_0} \times 100$

ii) Weighted Aggregative

→ Laspeyre's Index $= \frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times 100$

→ Paasche's Index $= \frac{\Sigma Q_1 P_1}{\Sigma Q_0 P_1} \times 100$

→ Fisher's Index $= \sqrt{\frac{\Sigma Q_1 P_0}{\Sigma Q_0 P_0} \times \frac{\Sigma Q_1 P_1}{\Sigma Q_0 P_1}} \times 100$

iii) Simple Average of Relatives

$$Q_{01} = \frac{\Sigma Q}{n} = \frac{1}{n} = \Sigma \left[\frac{q_1}{q_0} \times 100\right]$$

$$Q_{01} = \text{Antilog}\left[\Sigma \log \frac{Q}{n}\right]$$

iv) Weighted Average of Relatives

$$Q_{01} = \frac{\Sigma Q w}{w} = \left[\frac{\Sigma (q_1/q_0 \times 100) w}{\Sigma w}\right]$$

General Index from Group Indices

The General Index is the AM of the group Indices.

$$\text{General Index} = \frac{\Sigma W I}{\Sigma W}$$

Where,

W = Group Weight

I = Group Index,

EXERCISE

Time Series & Moving Average

A collection of data recorded over a period of time is called a time series.

Components of Time Series

- Trend (T)
- Cyclical Fluctuation (C)
- Seasonal Variation (S)
- Irregular Movements (I)

Observed Value (Y)

Additive Method: $Y = T + C + S + I$

Multiplicative Method: $Y = TSCI$.

Smoothening of Time Series Using Moving Average.

$$\boxed{\text{Moving Average} = \frac{\text{Observed Value}}{T \times C}}$$

$$\boxed{SI = \frac{TCSI}{T \times C}.}$$

EXERCISE

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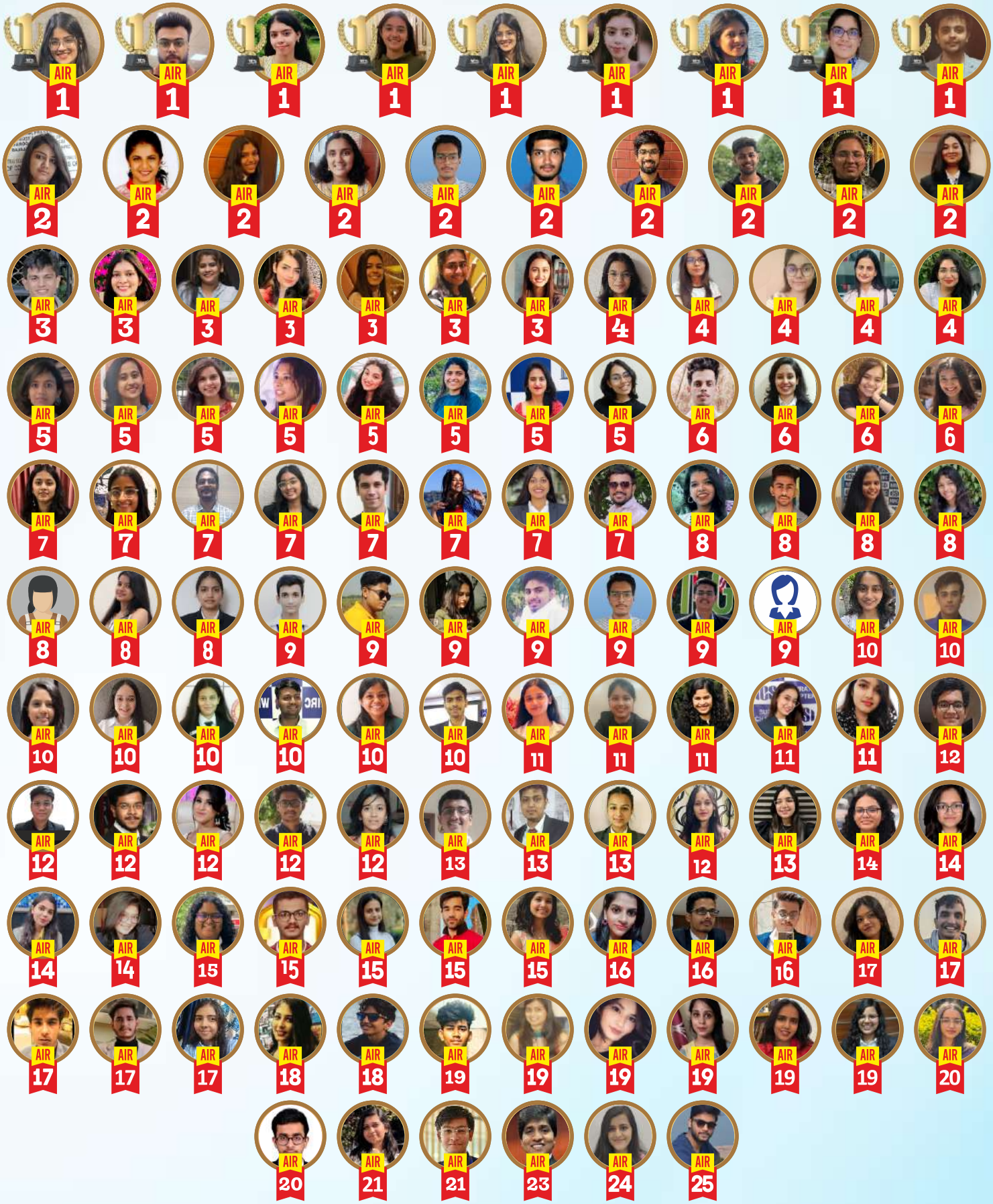

& many more

CS VAIBHAV CHITLANGIA

Vaibhav Chitlangia is a law graduate from ILS Law College, Pune, and a Company Secretary with All India Ranks at all three levels of the program. He secured All India Rank 15 in the Foundation Programme (June 2016), All India Rank 22 in the Executive Programme (June 2017), and All India Rank 4 in the Professional Programme (June 2018). In addition, he holds an LLM degree with specialization in Corporate Laws, a Diploma in Cyber Laws, and a Post Graduate Diploma in Medical Law and Ethics.

Known for his intellectual brilliance and tireless work ethic, Vaibhav ensures the success of his students by providing them with the best learning experience possible. His teaching style is engaging and student-centric. Vaibhav is known for making complex concepts easy to understand, using real-world examples and practical scenarios to bridge the gap between theory and practice. His method includes breaking down difficult topics into bite-sized chunks, allowing you to grasp each concept thoroughly before moving on to the next. This approach not only boosts your confidence but also helps you apply what you've learned effectively.

Vaibhav has also gained invaluable experience working with AZB & Partners, one of India's leading law firms. abroad. Since 2018, he has been guiding CA, CS, and Law students and has taught students across the country.

He strongly believes that "The only impediment in the path of success is a person's own mindset; if that is controlled, every feat is achievable."

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